Contracting with Private Rewards*

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Abstract

The canonical moral hazard model is extended to allow the agent to face endogenous and non-contractible uncertainty. The agent works for the principal and simultaneously pursues outside rewards. The contract offered by the principal thus manipulates the agent's work-life balance. The participation constraint is slack whenever it is optimal to distort the agent's work-life balance away from life compared to a symmetric information benchmark. Then, the agent's expected utility is high and he faces flatter incentives. Such contracts may be optimal when the two activities are strong substitutes in the agent's cost function or when reservation utility is low.

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1 Introduction

The principal-agent model has been tremendously influential in economics. However, by treating leisure as an exogenous black box, the canonical model in many ways ignores that a rich part of the agent's life takes place outside the office. In reality, workers are not just passive consumers when they are off the job. For instance, Bryson and MacKerron (2017) document that the leisure activities that people are the most happy engaging in all involve physical or mental exertion.[1] More broadly, non-work activities include home production, see e.g. Becker (1965). The agent devotes time and attention to a range of activities from the mundane to the extraordinary, like a complicated do-it-yourself renovation. Moonlighting represents yet another relevant activity.

Hence, it is more accurate to think of the agent as multi-tasking: He invests effort in two arenas, summarized here as "work" and "life". In this paper, the latter more broadly refers to whatever activities the agent is pursuing when he is not working for the principal (leisure, home production, moonlighting).

This paper proposes a multi-tasking model in which the agent decides how hard to work on the job and how hard to pursue non-contractible outside rewards. Rewards from the two sources are substitutes. Likewise, effort devoted to the two activities are substitutes in the agent's cost function. A key point then is that the contract manipulates the agent's entire work-life balance. This is enough to twist familiar intuition, with important consequences.

For instance, under normal circumstances the principal can calibrate a reduction in the wage schedule in a way that does not alter incentives on the job, although doing so may discourage participation. However, in the current model declining wages spur the agent to substitute effort towards "life" and away from "work." Similarly, a steepening of the wage schedule is normally taken to imply stronger incentives for work effort. While this direct effect still exists, there is now an additional countervailing indirect effect. Specifically, a steep wage schedule implies more risk, which causes the job to become less attractive to the risk averse agent. This effect again encourages a substitution of effort away from work. The

[1] Examples include going to a concert, exercising, gardening, hiking, pursuing a hobby, etc. Passive activities such as watching TV or browsing the internet are much further down the list.

latter effect dominates if the two activities are strong substitutes. Steepening the contract is then self-defeating as effort on the job declines.

The principal cares about work-life balance to the extent that it influences the cost of incentivizing work. It may be optimal to keep the wage schedule relatively flat. With less risky labour income, the incentive to supplement with private rewards diminishes. As the agent is less distracted by life, he may now increase effort at work despite the flatter wage schedule. The average wage can then be decreased. However, as previously explained, wages cannot be reduced too much without destroying incentives. Thus, the agent may earn rent in excess of his reservation utility. In summary, (i) the wage schedule is relatively flat and (ii) the agent earns high expected utility. Much of the paper is devoted to formalizing this intuition and understanding when these features are optimal.

These properties set the model apart from more standard models and may help resolve some common criticisms of the latter. First, standard models predict a binding participation constraint. This is arguably somewhat puzzling in light of amble evidence that employed people are happier than unemployed people, see e.g. Clark and Oswald (1994). Second, a common objection is that real world incentives appear weaker than what the standard model suggests.

Englmaier and Leider (2012) develop a reciprocity-based model in which the agent becomes "intrinsically motivated" to repay the principal's kindness when he is offered a generous contract. Thus, flatter extrinsic incentives are needed and they go hand in hand with higher expected utility. The current paper's explanation is instead based on endogenizing the agent's work-life balance.

The formal analysis requires an examination of the interplay between participation and incentive constraints. The symmetric information benchmark is a useful starting point. Here, the principal can dictate effort towards both work and life all while offering a fixed wage. Given the wage and work effort, it is optimal to permit the agent to pursue the level of life that maximizes his utility. The participation constraint can then be satisfied at the lowest possible wage.

Under asymmetric information, however, the participation constraint interacts with the incentive constraint on life. At low levels of life, the job must be made more attractive in order to compensate for low private rewards and secure participation. This is consistent with disincentivizing life because private rewards are less of a draw when the job is attractive. Hence, the two constraints are compatible. Note that a job with a low expected wage may still be attractive to the agent if it entails very little risk. Under reasonable assumptions, the incentive constraint is the stricter constraint. Thus, the only way to entice the agent to reduce his level of life below the symmetric information level is by granting him utility above reservation utility. In contrast, the two constraints conflict when the aim is to induce the agent to aggressively pursue life. Participation requires very high rewards to compensate for high effort costs, but this tends to disincentivize life. In some cases, it is impossible to write a contract that resolves this conflict.

The purpose of distorting life is to minimize the cost of incentivizing work. There is a trade-off. Pushing the agent to lower his pursuit of private rewards tightens the incentive constraint on life as he must be persuaded to divest from life. However, this leads to a weakening of the incentive constraint on work as the agent's marginal cost of effort is reduced. The second effect is stronger when the two activities are strong substitutes. Then, it is optimal to skew the agent's work-life balance away from life. Flat incentives now serve a dual purpose. First, they make the job attractive. Wages can then be lowered to the principal's benefit while still disincentivizing life. Second, since the agent is not pursuing life too hard, flat incentives are sufficient to incentivize work effort. Of course, there is also a limit to how flat the wage schedule can be made without destroying incentives for work. The second-best contract is exactly such that a perturbation that makes it marginally steeper or flatter has no first-order effect on work effort.

The participation constraint is more likely to be slack the lower reservation utility is. Then, the symmetric information level of life is high because a low wage is enough for participation but this spurs the agent to seek private rewards. Marginal costs are therefore high and it is more likely to be beneficial to disincentivize life. However, this necessitates giving the agent supernormal rents.

Work-life balance is a worthy topic of study in its own right. Nevertheless, the standard one-task model ignores the issue. Similarly, the dominant multi-tasking model is structured too rigidly to address the topic in a completely satisfactory way. The Linear-Exponential-Normal (LEN) model due to Holmström and Milgrom (1987, 1991) can allow for private rewards but only in such a way that there is essentially no interaction between rewards. As a result, for any given level of effort on the job, the principal is unable to manipulate how hard the agent pursues private rewards. Thus, in this application the LEN model is hardly a multi-dimensional model after all. The current model is more flexible and provides a more nuanced understanding of work-life balance.

Holmström and Milgrom (1991) consider a teacher who teaches both "basic skills" and "higher-order thinking skills." Basic skills can be tested, thus yielding a contractible signal. On the other hand, although they cannot be measured or directly rewarded, the teacher may experience some degree of satisfaction from teaching higher-order skills. Holmström and Milgrom (1991) argue that good test scores should be rewarded less than might be expected. The reason is that the only way to induce more effort on teaching higher-order skills in their model is to disincentivize the teaching of basic skills. Basic skills must be sacrificed for higher-order skills. This is not the case in the current model.

Thus, the two models yield flat incentives for different reasons and under different circumstances. In Holmström and Milgrom (1991), flat incentives discourage effort on the task that yields a contractible signal and the agent then substitutes effort towards the other task. In the current model, flatter incentives can also be used when the principal wishes to maintain a constant effort on the former task but to independently induce lower effort on the latter task.

The analysis combines ideas from Grossman and Hart (1983) with techniques used in the first-order approach, see Rogerson (1985), Jewitt (1988), Conlon (2009), and Kirkegaard (2017). Only the latter allows multi-tasking. The technical contribution is to extend these methods to multi-tasking with private rewards.

2 Model and preliminaries

2.1 The problem

The agent performs two "tasks", a_1 and a_2. For simplicity and unless a statement is made to the contrary, assume that $a_i \in [\underline{a}_i, \infty)$, $i = 1, 2$. A bounded domain is discussed in Section 5. The first task, a_1, captures the agent's effort on the job, as a result of which a contractible signal, x_1, is produced. The signal's marginal distribution is $G^1(x_1|a_1)$. The second task, a_2, reflects the agent's pursuit of a private reward. The agent receives a possibly non-monetary reward, x_2, which is determined by the marginal distribution function $G^2(x_2|a_2)$. Assume x_i belongs to a compact interval, $[\underline{x}_i, \overline{x}_i]$, which is independent of a_i. Let $g^1(x_1|a_1)$ and $g^2(x_2|a_2)$ denote the densities and assume that $g^i(x_i|a_i) > 0$ for all $x_i \in [\underline{x}_i, \overline{x}_i]$ and all $a_i \in [\underline{a}_i, \infty)$.[2] Note that each marginal distribution depends only on one task. This is further strengthened by assuming that x_1 and x_2 are independent.

Assumption A1 (Independence): *Outcomes are independent*, i.e. given a_1 and a_2, the joint distribution is given by

$$F(x_1, x_2|a_1, a_2) = G^1(x_1|a_1)G^2(x_2|a_2). \tag{1}$$

Independence implies that results are driven solely by the interactions in the agent's utility function. There are no confounding effects from a motive to manipulate the dependence structure. Moreover, there is no reason to think that the quality of the foliage on the agent's hike is correlated with how lucky he is on the job. Likewise, the agent's luck on the job is unlikely to influence how many riders he gets on the weekend when he drives for Uber.

A contract is a function $w(x_1)$ that specifies the wage paid to the agent for any signal realization. If the agent takes action (a_1, a_2) and the rewards on and outside the job are w and x_2, respectively, then the agent's Bernoulli utility is

$$v(w, x_2) - c(a_1, a_2), \tag{2}$$

where v is a rewards function and c is a cost function. Thus, given the contract $w(x_1)$, the agent's expected payoff from action (a_1, a_2) is

$$EU(a_1, a_2|w(\cdot)) = \int\int v(w(x_1), x_2)g^1(x_1|a_1)g^2(x_2|a_2)dx_1dx_2 - c(a_1, a_2). \tag{3}$$

The principal is risk neutral. He derives some direct benefit from the agent's action. For clarity, it is first assumed that he only cares directly about a_1. Hence, a_2 is of interest to the principal only insofar as it can be manipulated to minimize

[2] Throughout, all exogenous functions are assumed continously differentiable to the requisite degree. For brevity, statements to that effect are omitted from the numbered assumptions. A detailed discussion of assumptions can be found in the Online Appendix.

the cost of implementing a_1. The benefit function is denoted $B(a_1)$. Expected wage costs if the agent is induced to take action (a_1, a_2) are denoted $E[w|a_1, a_2]$.

Assumption P1 (The principal's preferences): The principal is risk neutral, with expected utility $B(a_1) - E[w|a_1, a_2]$.

The principal's problem is to maximize his expected utility, subject to the participation constraint (P) and the incentive compatibility constraint (IC), or

$$\max_{a_1,a_2,w} B(a_1) - \int w(x_1)g^1(x_1|a_1)dx_1$$

$$st. \quad EU(a_1, a_2|w(\cdot)) \geq \overline{u} \tag{P}$$

$$(a_1, a_2) \in \arg \max_{(a_1',a_2') \in [\underline{a}_1,\overline{a}_1]\times[\underline{a}_2,\overline{a}_2]} EU(a_1', a_2'|w(\cdot)), \tag{IC}$$

where $\overline{u}$ is the agent's reservation utility. Any action that solves the problem is referred to as a second-best action.

The problems faced by the two parties have now been outlined in broad terms. However, more specific assumptions are required to solve these problems and generate economic insights. Thus, to continue, define $l^i(x_i|a_i) = \ln g^i(x_i|a_i)$. Let $l^i_{a_i}(x_i|a_i)$ denote the likelihood-ratio, i.e. the derivative of $l^i(x_i|a_i)$ with respect to a_i, $i = 1, 2$, and assume it is bounded. The next assumption is standard.

Assumption A2 (MLRP): The marginal distributions have the (strict) *monotone likelihood ratio property*, i.e. for all $a_i \in [\underline{a}_i, \overline{a}_i]$ it holds that

$$\frac{\partial}{\partial x_i}\left(l^i_{a_i}(x_i|a_i)\right) = \frac{\partial^2 \ln g^i(x_i|a_i)}{\partial a_i \partial x_i} > 0 \text{ for all } x_i \in [\underline{x}_i, \overline{x}_i]\,. \tag{4}$$

Rogerson (1985) considers a one-signal, one-task model and assumes that the distribution function is convex in the one-dimensional action. Kirkegaard (2017) allows multiple tasks and signals and assumes that the distribution function is convex in the many-dimensional action. The same assumption is useful here.

Assumption A3 (LOCC): $F(x_1, x_2|a_1, a_2)$ satisfies the *lower orthant convexity condition*; $F(x_1, x_2|a_1, a_2)$ is weakly convex in (a_1, a_2) for all (x_1, x_2) and (a_1, a_2).

Assumptions A1–A3 describe the agent's "technology". His preferences are described by Bernoulli utility of the form in (2). The rewards function $v(w, x_2)$

is strictly increasing and strictly concave in each argument, $v_i > 0 > v_{ii}$, $i = 1, 2$, where subscripts denote derivatives.[3] The domain is $\mathbb{R} \times [\underline{x}_2, \overline{x}_2]$. The assumption that $w \in \mathbb{R}$ ensures that wages are interior. Thus, wages may be negative.

Rewards and tasks are substitutes. Thus, assume that $v_{12} < 0$; the higher x_2 is, the lower is the marginal utility of additional employment income. Assume also that $v_2(w, x_2) \to 0$ as $w \to \infty$. Thus, the marginal benefit of additional private rewards vanishes as the agent's labor income becomes unboundedly high. It should be acknowledged that rewards or actions are sometimes complements in the real world. The model does not cover such situations.

Likewise, a_1 and a_2 are weak substitutes in the cost function, or $c_{12} \geq 0$. That is, the marginal cost of increasing a_1 is higher when a_2 is high. The assumption is satisfied in e.g. Holmström and Milgrom's (1991) leading model, the effort and attention allocation model, where costs depend only on $a_1 + a_2$. The cost function is strictly increasing and jointly convex in (a_1, a_2).

Assumption A4 (Preferences): The agent's Bernoulli utility is $v(w, x_2) - c(a_1, a_2)$; $v(w, x_2)$ is strictly increasing and strictly concave in each argument, with domain $\mathbb{R} \times [\underline{x}_2, \overline{x}_2]$, while $c(a_1, a_2)$ is strictly increasing and weakly convex in (a_1, a_2). Rewards are strict substitutes; $v_{12}(w, x_2) < 0$ with $v_2(w, x_2) \to 0$ as $w \to \infty$. Tasks are weak substitutes; $c_{12}(a_1, a_2) \geq 0$.

2.2 Preliminary observations

At this stage, the properties of the endogenous contract are unknown. To get a feel for the problem, however, it is useful to begin by considering the agent's problem if the contract $w(x_1)$ is differentiable and strictly increasing.

Definition (regular contracts): The contract is said to be *regular* if it is differentiable and strictly increasing, with $w'(x_1) > 0$ for all $x_1 \in [\underline{x}_1, \overline{x}_1]$.

Now, $EU(a_1, a_2|w(\cdot))$ is strictly concave when the contract is regular. In this case, the agent's optimal action is unique. All proofs are in the Appendix.

[3] It is not necessary for $v(w, x_2)$ to be jointly concave in (w, x_2) in order for the agent's expected utility to be concave. See Lemma 1, below, and its proof. For comparison, in Rogerson (1985) concavity of the Bernoulli utility function is used only to prove that the contract is monotonic. Beyond that, it is not invoked to prove that the agent's problem is concave.

Lemma 1 *Assume that A1–A4 hold. For any regular contract, EU is strictly concave in* (a_1, a_2)*, with* $EU_{11} < 0$*,* $EU_{22} < 0$*,and* $EU_{11}EU_{22} - EU_{12}^2 > 0$*. Moreover, the two tasks are strict substitutes, or* $EU_{12} < 0$*.*

To compare the agent's problem to a more standard problem, define

$$V(w, a_2) = \int v(w, x_2) g^2(x_2|a_2) dx_2 \tag{5}$$

as the expected utility of a fixed wage, given that the agent exerts effort a_2 towards private rewards. Note that the expectation is over x_2, given a_2. Then,

$$EU(a_1, a_2|w(\cdot)) = \int V(w(x_1), a_2) g^1(x_1|a_1) dx_1 - c(a_1, a_2). \tag{6}$$

If a_2 is exogenous, the expression in (6) is identical to expected utility in a standard model where Bernoulli utility and costs are parameterized by a_2. In this case, it is well known that (P) must bind if the contract $w(x_1)$ is optimal. Otherwise, construct another contract, $\widehat{w}_\varepsilon(x_1)$, such that $V(\widehat{w}_\varepsilon(x_1), a_2) = V(w(x_1), a_2) - \varepsilon$ for all x_1, with $\varepsilon > 0$. The two contracts induce the same a_1 and both satisfy (P) if ε is small enough. Since $\widehat{w}_\varepsilon(x_1)$ entails lower wages, $w(x_1)$ cannot be optimal.

The problem is more complex when a_2 is endogenous. The agent *jointly* chooses a_1 *and* a_2. As always, there is a direct incentive effect on a_1 from changes in the contract. There is also an indirect effect that comes from the accompanying change in a_2. Simply put, an increase in a_2 tends to trigger a substitution away from a_1. Contract changes may now have unexpected implications.

First, consider shifting the wage schedule down from $w(x_1)$ to $\widehat{w}_\varepsilon(x_1)$. The job is now less rewarding and attention shifts towards pursuing private rewards. The implication is that reducing wages makes it harder to prevent life incentives. Then, as private rewards increase, the incentive to pursue labor rewards lessens. Due to this "substitution effect," a_2 increases and a_1 decreases.

Proposition 1 *Assume that A1–A4 hold. Fix a regular contract,* $w(x_1)$*, and let* (a_1^*, a_2^*) *denote the interior action that it induces. If the agent participates, let* (a_1', a_2') *denote the action that is induced when* $w(x_1)$ *is replaced by* $\widehat{w}_\varepsilon(x_1)$ *where* $V(\widehat{w}_\varepsilon(x_1), a_2^*) = V(w(x_1), a_2^*) - \varepsilon$*,* $\varepsilon > 0$*. Then,* $a_1' < a_1^*$ *and* $a_2' > a_2^*$*.*

Proposition 1 demonstrates that the argument used to show that (P) binds in a standard model breaks down when a_2 is endogenous. The new incentive constraint interferes. Indeed, a key point of the paper is that (P) may not bind.

Second, consider replacing $w(x_1)$ by some other regular contract, $\widehat{w}_\alpha(x_1)$, that has the same expected wage at effort $a_1 = a_1^*$. Moreover, assume that $\widehat{w}_\alpha(x_1)$ is greater than $w(x_1)$ when the likelihood-ratio $l^1_{a_1}(x_1|a_1^*)$ is positive and smaller when the likelihood-ratio is negative. Thus, the two contracts cross once and $\widehat{w}_\alpha(x_1)$ is in some sense steeper. At a_1^*, the distribution of wages under the new contract is a mean-preserving spread over wages under the original contract. Were the agent to keep constant his action at (a_1^*, a_2^*), the additional risk would lower his expected utility. However, the increased risk impacts incentives for two reasons. Most directly, it encourages higher effort on the job, as in a standard model. On the other hand, the added risk makes the job a worse source of rewards and the agent therefore also has an added incentive to increase his pursuit of private rewards. This again creates an incentive to substitute away from effort on the job. Due to this indirect effect, it is ambiguous whether a_1 increases or decreases.

Thus, it is possible that steepening the contract is *self-defeating* and leads the agent to lower a_1. Intuitively, this is more likely when the tasks are strong substitutes. Since $c_{12} > 0$, the increase in a_2 forces c_1 higher, thus diminishing incentives for a_1. This effect is more pronounced the higher c_{12} is. On the other hand, incentives for a_2 depend on c_2. If c_{22} is small, then c_2 is not very responsive and a small contract change may then trigger a large change in a_2, which may then in turn have a larger impact on a_1. In sum, the equilibrium response is more likely to be a lowering of a_1 when c_{12} is large and c_{22} is small.

The next result formalizes this intuition under the additional assumption that $V_{112}(w, a_2) > 0$. That is, the harder the agent pursues private rewards, the "less negative" V_{11} is, suggesting that the agent in a sense becomes less risk averse.

Proposition 2 *Assume that A1–A4 hold and that $V_{112}(w, a_2) > 0$. Fix a regular contract, $w(x_1)$, and let (a_1^*, a_2^*) denote the interior action that it induces. If the agent participates, let (a_1', a_2') denote the action that is induced when $w(x_1)$ is replaced by the contract $\widehat{w}_\alpha(x_1)$ described above. Then, $a_1' < a_1^*$ and $a_2' > a_2^*$ if $c_{12}(a_1, a_2)$ is sufficiently large relative to $c_{22}(a_1, a_2)$.*

Propositions 1 and 2 suggest that it may be optimal to flatten the wage schedule. When c_{12} is large enough, the agent then increases a_1. The new and flattened wage schedule can then be shifted down to bring a_1 back down to a_1^*. This last step lowers implementation costs. However, Proposition 1 also implies that there is a limit to how much the wage can be reduced without destroying incentives. Thus, it is possible that (P) may be slack. On the other hand, when c_{12} is small the principal can shift down the wage schedule but reincentivize effort on the job with a steeper wage schedule. The principal may be able to continue in this manner until (P) becomes binding. Thus, to be clear, there are environments where (P) binds. However, since the novelty is that there are circumstances under which (P) is slack and the contract is relatively flat, much of the paper naturally focuses on pursuing and explaining this possibility.

The intuition behind Propositions 1 and 2 and the interplay between c_{12} and c_{22} reappear later in the paper. Formally, however, these results are not without their weaknesses. First, they assume that contracts are regular. Second, they start from an arbitrary incentive compatible contract and then consider specific types of contract changes. The rest of the paper studies optimal contracts and verifies that the intuition is robust. The link between the agent's work-life balance and the participation constraint under the second-best contract is also clarified.

2.3 Towards optimal contracts

Generalizing an idea due to Grossman and Hart (1983), a three-step procedure is used to attack the problem. The first step derives the cheapest contract that induces any fixed (a_1, a_2) action. The second step holds fixed effort on the job, a_1, and minimizes implementation costs with respect to a_2. The third step maximizes the principal's net payoff over a_1. Note that the benefit function $B(a_1)$ is irrelevant for the first two steps. The first step is conceptually and technically the most challenging. Several of the paper's economic insights are derived from the second step. The third step is less interesting and is largely ignored.

Consider the first step. A good place to start is by examining interior actions first. For such actions, incentive compatibility necessitates that expected utility achieves a stationary point at that action, or $EU_1 = 0 = EU_2$. These constraints are referred to as the "local" incentive compatibility constraints. The shorthand L-IC$_i$ is used to refer to the constraint that $EU_i = 0$, $i = 1, 2$, while L-IC refers to L-IC$_1$ and L-IC$_2$ together. Now imagine minimizing wage costs subject to (P) and L-IC. If the resulting solution is a regular contract, then the agent's problem is concave and L-IC is indeed sufficient for incentive compatibility. A key technical challenge is thus to establish that the candidate solution is regular.

Thus, holding fixed the action, consider the solution to the above problem. Let $\lambda \geq 0$ denote the multiplier on the participation constraint. Let μ_1 and μ_2 denote the multipliers on L-IC$_1$ and L-IC$_2$, respectively. The optimal wage if x_1 is observed is implicitly characterized by the necessary first order condition

$$\lambda + \mu_1 l^1_{a_1}(x_1|a_1) = \frac{1}{V_1(w(x_1), a_2)} - \mu_2 \frac{V_{12}(w(x_1), a_2)}{V_1(w(x_1), a_2)}. \tag{7}$$

The last term in (7) generally complicates the analysis. To begin, a special model specification in which this complication is minimized is considered.

Definition (The multiplicative model): In the *multiplicative model*, the rewards function is

$$v(w, x_2) = -m(w)n(x_2), \tag{8}$$

where m and n are strictly *negative* functions that are strictly increasing and strictly concave.

The unusual signs are chosen so that rewards are substitutes in the multiplicative model and not complements. Imagine private rewards are monetary and that the agent has constant absolute risk aversion over total income. Then, utility from rewards is $v(w, x_2) = -e^{-r(w+x_2)}$, $r > 0$. This fits the multiplicative model, with $m(w) = -e^{-rw}$ and $n(x_2) = -e^{-rx_2}$. The agent's expected utility now takes the convenient form

$$EU(a_1, a_2|w(\cdot)) = -\left(\int m(w(x_1))g^1(x_1|a_1)dx_1\right)\left(\int n(x_2)g^2(x_2|a_2)dx_2\right) - c(a_1, a_2).$$

The first factor can be interpreted as the agent's expected utility from labor income and the second factor as expected utility from private rewards. For brevity, denote the former $M(a_1|w(\cdot))$ and the latter $N(a_2)$, such that

$$EU(a_1, a_2|w(\cdot)) = -M(a_1|w(\cdot))N(a_2) - c(a_1, a_2).$$

Throughout the paper, the analysis relies on a fundamental relationship between incentive compatibility, particularly L-IC$_2$, and the participation constraint. This relationship is especially strong in the multiplicative model. Here, if the interior action (a_1, a_2) is to be implemented, L-IC$_2$ requires that

$$-M(a_1|w(\cdot))N'(a_2) - c_2(a_1, a_2) = 0,$$

and only $M(a_1|w(\cdot))$ can be manipulated to achieve this. Since utility from labor income is now pinned down by L-IC$_2$, the principal has no more degrees of freedom with which to manipulate the agent's expected utility, holding fixed the action. Hence, expected utility at the target action is already *fully determined*, with

$$EU(a_1, a_2|w(\cdot)) = \frac{N(a_2)}{N'(a_2)}c_2(a_1, a_2) - c(a_1, a_2). \tag{9}$$

Thus, given the fixed action and an incentive compatible contract, it would be purely coincidental if (P) happens to bind. In particular, if (9) exceeds $\overline{u}$ then the participation constraint is *redundant* as it is implied by incentive compatibility. Conversely, if (9) is below $\overline{u}$ then (P) must necessarily be violated. In this case, the action is not implementable as the agent would refuse the contract. In fact, it can be verified that expected utility as derived in (9) is decreasing in both levels of a_1 and a_2 that are to be implemented. Hence, given $\overline{u}$, (P) is redundant if a_1 and a_2 are small enough but it is violated if they are large enough. The reason behind this phenomenon is explained in the next section.

Turning now to the shape of the optimal contract, a distinguishing feature of the multiplicative model is that $V(w, a_2) = -m(w)N(a_2)$ and so the last term in (7) reduces to $N'(a_2)/N(a_2)$. Since this is independent of w, (7) is virtually identical to its counterpart in a standard model. Standard arguments then apply. Thus, $\mu_1 > 0$ and the contract is regular; see Rogerson (1985, footnote 8) or Jewitt (1988, Lemma 1). This confirms that the optimal contract that induces the action in question – provided it is implementable – does indeed take the form in (7). In principle, the two binding constraints, L-IC$_1$ and L-IC$_2$, can be used to solve for μ_1 and μ_2. Since (P) is redundant if the action is implementable, $\lambda = 0$.

The following example uses the multiplicative model to illustrate several of the paper's main findings. Details are in the Online Appendix.

EXAMPLE 1: Assume that $a_i \in [0,1]$, $i = 1, 2$, and that

$$G^i(x_i|a_i) = \left(1 - e^{-\frac{a_i+8}{72}}\right) x_i^2 + \left(e^{-\frac{a_i+8}{72}}\right) x_i, \quad x_i \in [0,1].$$

It can be verified that Assumptions A1–A3 are satisfied. Assume also that

$$c(a_1, a_2) = 0.7313a_2 + 0.405a_1^2 + 0.005a_2^2 + 0.09a_1a_2.$$

For computational ease, this example is constructed in such a way that $c_{ij}(a_1, a_2)$ is constant, $i, j = 1, 2$. Likewise, $c_{11}c_{22} - c_{12}^2 = 0$. Finally, the rewards function is

$$v(w, x_2) = -\left(2\sqrt{w} - k\right)(7.5\sqrt{x_2} - 8),$$

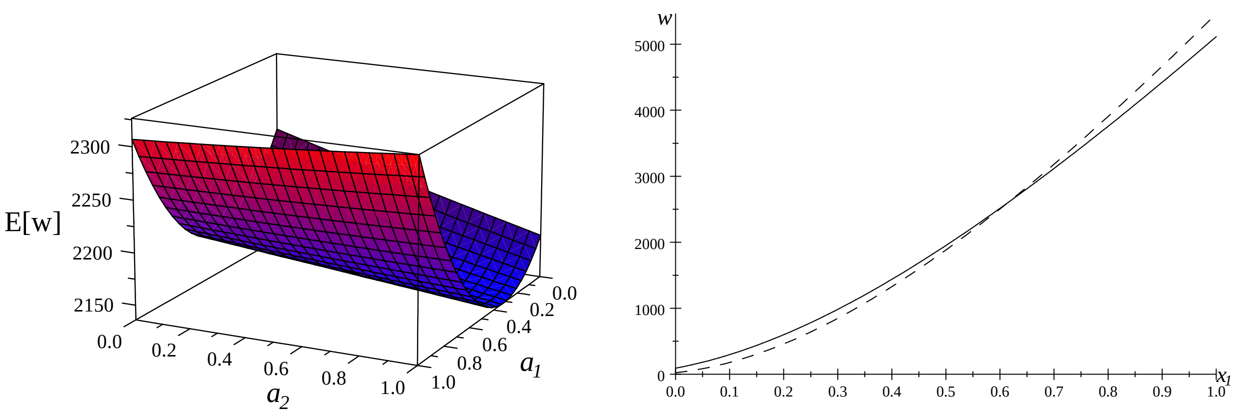
where $w \in [0, (\frac{k}{2})^2]$, $x_2 \in [0,1]$ and $k = 153.696$. Assumption A4 is satisfied, with the exception that w cannot be any real number. The restriction that $w \geq 0$ is simply to ensure that $v(w, x_2)$ is well-defined. The role of k is to ensure that $m(w) = 2\sqrt{w} - k$ is strictly negative as required in the multiplicative model. Thus, the exact value of k has been chosen to ensure that any optimal contract features strictly positive wages and that $m(w) < 0$. It is for similar reasons that a_i is bounded above in this example. Note that since optimal wages are strictly positive, results are not driven by a limited liability constraint.

The example exhibits a number of interesting properties.

Locally increasing or decreasing implementation costs: Figure 1(a) depicts implementation costs for interior actions, assuming they are all implementable (i.e. ignoring the participation constraint). Implementation costs are decreasing in a_2 when a_1 is small but increasing in a_2 when a_1 is large. There is an intermediate range of a_1 where implementation costs are u-shaped in a_2. Thus, the optimal level of a_2 depends on a_1 and it may or may not be small. Finally, implementation costs are u-shaped in a_1, given a_2. The intuition is explained in Section 4.

Slack participation constraint: To continue towards the second step of the principal's problem – finding the optimal a_2 given a_1 – imagine that he wants to induce $a_1 = 0.9$. As a point of reference, (9) implies that any contract that induces action $(a_1, a_2) = (0.9, 0.9)$ gives the agent expected utility of -194. Thus, if $\overline{u} = -194$, any $(a_1, a_2) = (0.9, a_2)$ with $a_2 \leq 0.9$ can be induced. In this case, the optimal a_2 to induce alongside $a_1 = 0.9$ is clearly $a_2 = 0$ since this is where implementation costs are minimized; see Figure 1(a).[4] Here, the agent's expected utility is $-189.5 > -194$, by (9).

Weaker incentives: In this example, the optimal contracts that induce the same a_1 but different a_2 cross each other at most once. The optimal contract that induces the lower a_2 crosses the other one from above and thus features wages that are more "compressed." For example, the optimal contract that induces $(a_1, a_2) = (0.9, 0.9)$ is depicted as the dashed curve in Figure 1(b) alongside the optimal contract that more cheaply induces $(0.9, 0)$ (the unbroken curve). Intuitively, the more compressed contract carries less risk and is therefore more attractive. Since work is now a good source of utility, the agent relaxes his pursuit of outside rewards. In fact, in this particular example the more compressed contract also features a lower expected wage.



(a) Implementation costs. (b) Comparing contracts.

Figure 1: Implementation costs and optimal contracts.

[4] Implementation costs as shown in Figure 1(a) are valid also along the $a_2 = 0$ boundary. Here, incentive compatibility requires $EU_2 \leq 0$. However, by using a version of the standard argument before Proposition 1, it can never be optimal to leave this constraint slack.

Incentives, utility, and work-life balance: Summarizing the above findings, the contract that minimizes the cost of implementing $a_1 = 0.9$ skews the agent's work-life balance away from life compared to the contract that gives the agent exactly reservation utility ($a_2 = 0$ instead of $a_2 = 0.9$). The agent faces weaker incentives and earns higher expected utility in the former case.[5] ▲

The remainder of the paper is devoted to further understand and generalize the points made by the example. Two issues are particularly important to clarify. The first is when and why (P) might be made redundant by incentive compatibility. Section 3 is devoted to this issue. The second is to characterize optimal contracts and to determine under which circumstances it is optimal to induce an action for which (P) is redundant. This is taken up in Section 4.

3 Participation versus incentive constraints

This section shows that there are actions for which the participation constraint is redundant even beyond the multiplicative model. An intuitive argument that ignores L-IC$_1$ is given first. Then, a stronger assumption on the rewards function is introduced which makes it possible to account for L-IC$_1$ as well.

3.1 Participation versus private rewards

Two benchmarks are examined. The first focuses on L-IC$_2$ and the second on (P), while ignoring all other constraints. This makes it possible to isolate the implications of the two constraints and to ultimately better compare them. Note that L-IC$_1$ is ignored throughout. Hence, it is as if a_1 is contractible.

First, consider a partial-information benchmark in which a_1 is observable and contractible but a_2 and x_2 are not. Thus, there is an incentive problem with respect to a_2. The participation constraint is ignored. To begin, assume a fixed-wage contract is used to incentivize a_2. The agent's utility is then concave in a_2. If an interior a_2 is to be induced, the wage must be chosen to satisfy the

[5] The contract can also be compared to the optimal contract if $a_2 = 0$ holds exogenously. Under exogenous a_2, (P) binds and the wage in this example simply decreases such that $v(w(x_1), x_2)$ and $V(w(x_1), a_2)$ shift down by a constant. However, this does not generalize.

agent's first-order condition. Letting $L(a_1, a_2)$ denote the wage that is incentive compatible with Life, the condition is that $V_2(L(a_1, a_2), a_2) - c_2(a_1, a_2) = 0$, or

$$EU_2(a_1, a_2 | L(a_1, a_2)) = 0, \quad a_2 \in (\underline{a}_2, \infty). \tag{10}$$

By concavity, $L(a_1, a_2)$ is unique when a_2 is interior. The assumption that $v_2(w, x_2) \to 0$ as $w \to \infty$ implies that $V_2(w, a_2) \to 0$ as $w \to \infty$, which in turn means that $L(a_1, a_2)$ exists. This is the only role this assumption plays. Without it, it may be impossible to incentivize low levels of a_2. It can be verified that $L(a_1, a_2)$ is strictly decreasing in a_2. This can be thought of as formalizing the idea that higher a_2 are incentivized when the job is a poor source of rewards. If $a_2 = \underline{a}_2$, any fixed wage for which $EU_2(a_1, \underline{a}_2 | w) \leq 0$ is incentive compatible. This holds for large wages and (10) just gives a lower bound. See Figure 2.

Second, consider the symmetric information benchmark where the principal can dictate the agent's action, (a_1, a_2). There is by definition no incentive problem here. Given the agent is risk averse, the cheapest way to ensure participation is to offer a fixed-wage contract. Let $W(a_1, a_2 | \overline{u})$ denote the wage so derived, with

$$V(W(a_1, a_2 | \overline{u}), a_2) - c(a_1, a_2) = \overline{u}. \tag{11}$$

Unlike $L(a_1, a_2)$, $W(a_1, a_2 | \overline{u})$ depends on, and is increasing in, $\overline{u}$. If V is bounded above, as in the case of CARA utility, then it may be impossible to find a wage that ensures participation. To proceed, it is innocuous to limit attention to a_1 for which there exists some a_2 such that (11) has a solution.

It is easy to verify that $W(a_1, a_2 | \overline{u})$ is strictly convex in a_2. Holding a_1 fixed, let $s(a_1 | \overline{u})$ denote the cost-minimizing level of a_2 to dictate. This is the unique value of a_2 that minimizes $W(a_1, a_2 | \overline{u})$. The first order condition is that

$$\frac{\partial W(a_1, a_2 | \overline{u})}{\partial a_2} = -\frac{EU_2(a_1, a_2 | W(a_1, a_2 | \overline{u}))}{V_1(W(a_1, a_2 | \overline{u}), a_2)}$$

equals zero at $a_2 = s(a_1 | \overline{u})$. By definition of $L(a_1, a_2)$, this means that

$$W(a_1, s(a_1 | \overline{u}) | \overline{u}) = L(a_1, s(a_1 | \overline{u})).$$

In other words, $s(a_1|\overline{u})$ is found where $W(a_1,a_2|\overline{u})$ and $L(a_1,a_2)$ coincide. See Figure 2. Hence, $a_2 = s(a_1|\overline{u})$ maximizes the agent's utility given the fixed wage $W(a_1, s(a_1|\overline{u})|\overline{u})$ and given that a_1 has been dictated. Thus, there is no real reason to dictate a_2 as the agent would voluntarily select the correct value.

To understand why $W(a_1,a_2|\overline{u})$ is u-shaped, begin by examining small a_2. Here, private rewards are small and therefore large labor rewards are needed to secure participation. Similarly, when a_2 is large, the costs of effort are large and once again the job must be richly compensated. For moderate a_2 and in particular for $a_2 = s(a_1|\overline{u})$, private rewards are reasonably high and costs reasonable low. Consequently, a lower wage is required to achieve reservation utility.

The relative magnitudes of a_1 and $s(a_1|\overline{u})$ reflect the agent's work-life balance under symmetric information. Two comparative statics are of interest. First, in Figure 2, an increase in $\overline{u}$ shifts $W(a_1,a_2|\overline{u})$ up, and with it the point of intersection with $L(a_1,a_2)$. That is, $s(a_1|\overline{u})$ is decreasing in $\overline{u}$. If $\overline{u}$ is very large, however, then $W(a_1,a_2|\overline{u})$ must be large and the agent is always happy to take a small pay-cut to be allowed to lower a_2. In this case, $s(a_1|\overline{u}) = \underline{a}_2$. The paper focuses on the more interesting case where $s(a_1|\overline{u})$ is interior, or $s(a_1|\overline{u}) > \underline{a}_2$.

Second, since a_1 and a_2 are substitutes, it follows directly that $s(a_1|\overline{u})$ is decreasing in a_1. Higher a_1 drives up the marginal costs of a_2 and makes it less attractive to pursue private rewards. Thus, it is in the agent's and therefore the principal's interest to lower a_2. In other words, a_1 and $s(a_1|\overline{u})$ move in opposite directions under symmetric information.

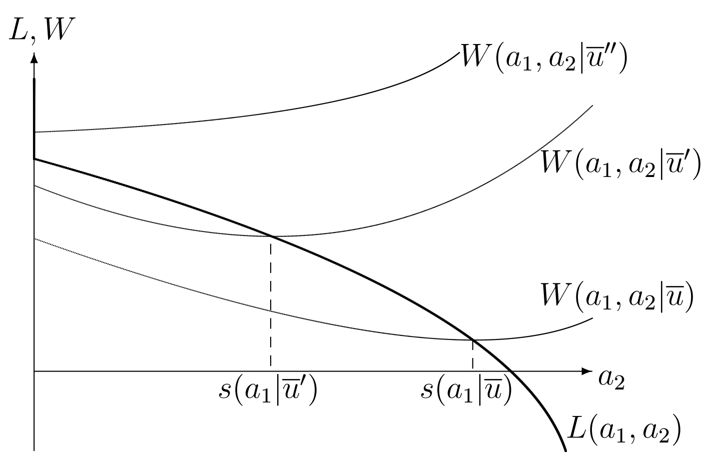


Figure 2: (P) and L-IC$_2$ with fixed-wage contracts, where $\overline{u}'' > \overline{u}' > \overline{u}$.

Returning to Figure 2, note that when $s(a_1|\overline{u})$ is interior,

$$W(a_1, a_2|\overline{u}) < L(a_1, a_2) \text{ for } a_2 < s(a_1|\overline{u}). \tag{12}$$

The two benchmarks share the properties that L-IC$_1$ is ignored and that fixed-wage contracts are used. In such settings, (12) implies that L-IC$_2$ imposes more of a constraint on the principal than does (P) when the agent's work-life balance is to be skewed away from life, or $a_2 < s(a_1|\overline{u})$. In fact, if both L-IC$_2$ and (P) are present simultaneously, the participation constraint is redundant when small a_2 are to be implemented with a fixed-wage contract. Incentive compatibility dictates a fixed wage of $L(a_1, a_2)$, but this gives the agent an expected utility that exceeds his reservation utility, which only requires a wage of $W(a_1, a_2|\overline{u})$. Hence, (P) can be ignored for such actions. Note that this holds even if $a_2 = \underline{a}_2$.

However, as attention shifts to the asymmetric information setting at the heart of the paper, L-IC$_1$ also comes into play. This constraint rules out fixed-wage contracts. Thus, the above argument is no longer quite strong enough to establish that (P) is redundant for fixed $a_2 < s(a_1|\overline{u})$. Conversely, for $a_2 > s(a_1|\overline{u})$ any fixed-wage contract that satisfies L-IC$_2$ must violate (P), but this need not be true for variable-pay contracts. The multiplicative model is a special case where the aforementioned conclusions hold for any type of contract. The reason is that L-IC$_2$ uniquely identifies the expectation of $m(w(\cdot))$, which is also what (P) relies on. Hence, in that model a_2 is implementable if and only if $a_2 \leq s(a_1|\overline{u})$.

Outside the multiplicative model, L-IC$_2$ nails down the expected value of $V_2(w(\cdot), a_2)$, yet this is not necessarily enough to determine the expectation of $V(w(\cdot), a_2)$. The resulting challenge is taken up next. Note, however, that even if $a_2 > s(a_1|\overline{u})$ is implementable, $W(a_1, a_2|\overline{u})$ of course still represents a lower bound on implementation costs. By convexity, $W(a_1, a_2|\overline{u})$ explodes as a_2 grows large and it follows that very high a_2 cannot be second-best.

3.2 Decreasing absolute risk aversion

Proposition 2 already suggests that $V_{112} > 0$ may play an important role in the analysis. In fact, a slightly stronger property turns out to be useful. To motivate this, a final but intuitive assumption is imposed on the agent's rewards function.

Assumption A5 (Log-supermodularity): The agent's marginal utility of labor income, $v_1(w, x_2)$, is log-supermodular in (w, x_2), or

$$\frac{\partial^2 \ln v_1(w, x_2)}{\partial w \partial x_2} \geq 0 \text{ for all } w \text{ and all } x_2 \in [\underline{x}_2, \overline{x}_2]. \tag{13}$$

Assumption A5 is equivalent to assuming that

$$\frac{\partial}{\partial x_2}\left(\frac{-v_{11}(w, x_2)}{v_1(w, x_2)}\right) \leq 0 \text{ for all } w \text{ and all } x_2 \in [\underline{x}_2, \overline{x}_2]. \tag{14}$$

Thus, the agent's absolute risk aversion over labor income is decreasing in x_2. That is, the agent is less sensitive to risk in labor income when the private reward is high. If the private reward is monetary and $v(w, x_2)$ takes the form $u(w + x_2)$, then Assumption A5 says that the agent has decreasing absolute risk aversion with respect to total income. In the multiplicative model, the inequality in (13) is an equality. Assumption A5 imposes structure on the function $V(w, a_2)$.

Lemma 2 *Given A1–A5, the function $V(w, a_2)$ has the following properties:*

1. *$V(w, a_2)$ is strictly increasing and strictly concave in both arguments,*

$$V_i(w, a_2) > 0 > V_{ii}(w, a_2), \quad i = 1, 2.$$

2. *w and a_2 are strict substitutes, $V_{12}(w, a_2) < 0$.*
3. *$V_1(w, a_2)$ is log-supermodular in (w, a_2),*

$$\frac{\partial}{\partial a_2}\left(\frac{-V_{11}(w, a_2)}{V_1(w, a_2)}\right) = \frac{\partial}{\partial w}\left(\frac{-V_{12}(w, a_2)}{V_1(w, a_2)}\right) \leq 0. \tag{15}$$

4. *The function $[-V_2(w, a_2)]$ is strictly increasing and strictly concave in w, and it has a weakly higher coefficient of absolute risk aversion than $V(w, a_2)$.*

Lemma 2 has two uses. Part 4 of the lemma is used in the next subsection to finish the argument that (P) is sometimes redundant. Part 3 implies that the last term in (7) is monotonic in wages. This in turn implies that the contract is regular if $\mu_1 > 0 \geq \mu_2$. Technically, then, the challenge becomes to verify that $\mu_1 > 0 \geq \mu_2$. This is pursued in Section 4.

Lemma 3 *Given A1-A5 and P1, $w(x_1)$ as defined in (7) is unique whenever $\mu_1 \geq 0 \geq \mu_2$. Moreover, $w(x_1)$ is regular if $\mu_1 > 0 \geq \mu_2$.*

3.3 Variable pay and the participation constraint

Fix some action that is to be induced, with $a_2 > \underline{a}_2$. L-IC$_2$ can be written as

$$\int \left[-V_2(w(x_1), a_2)\right] g^1(x_1|a_1)dx_1 = -c_2(a_1, a_2).$$

The last part of Lemma 2 suggests that the function $[-V_2(\cdot, a_2)]$ can be thought of as a pseudo-utility function. Then, use $[-V_2(\cdot, a_2)]$ to evaluate the certainty equivalent, $CE_{-V_2}(a_1, a_2|w(\cdot))$, of the contract, with

$$V_2(CE_{-V_2}(a_1, a_2|w(\cdot)), a_2) = \int \left[-V_2(w(x_1), a_2)\right] g^1(x_1|a_1)dx_1.$$

Now, L-IC$_2$ nails down this certainty equivalent; $CE_{-V_2}(a_1, a_2|w(\cdot))$ must exactly equal $L(a_1, a_2)$, by definition of the latter. Let $CE_V(a_1, a_2|w(\cdot))$ denote the certainty equivalent when evaluated using $V(\cdot, a_2)$. To ensure participation, this must be no smaller than $W(a_1, a_2|\overline{u})$. Hence, L-IC$_2$ and (P) are equivalent to

$$CE_{-V_2}(a_1, a_2|w(\cdot)) = L(a_1, a_2) \text{ and } CE_V(a_1, a_2|w(\cdot)) \geq W(a_1, a_2|\overline{u}).$$

By Lemma 2, $V(\cdot, a_2)$ is less affected by risk than $[-V_2(\cdot, a_2)]$. Hence, $V(\cdot, a_2)$ has a larger certainty equivalent than $[-V_2(\cdot, a_2)]$. Thus,

$$CE_V(a_1, a_2|w(\cdot)) \geq CE_{-V_2}(a_1, a_2|w(\cdot)) = L(a_1, a_2), \tag{16}$$

where the equality comes from L-IC$_2$. The point here is that L-IC$_2$ implies a lower bound on the utility experienced by $V(\cdot, a_2)$.

Next, assume that $s(a_1|\overline{u}) > \underline{a}_2$ and imagine inducing some $a_2 \in (\underline{a}_2, s(a_1|\overline{u})]$. From (12), $L(a_1, a_2)$ is no smaller than $W(a_1, a_2|\overline{u})$. Even with a fixed-wage contract, $V(\cdot, a_2)$ happily accepts a contract that is incentive compatible for

$[-V_2(\cdot, a_2)]$. Adding risk necessitates compensating $[-V_2(\cdot, a_2)]$ but this overcompensates the less risk averse $V(\cdot, a_2)$. Hence, $V(\cdot, a_2)$ still accepts the contract. Formally, combining (12) and (16) yields

$$CE_V(a_1, a_2|w(\cdot)) \geq CE_{-V_2}(a_1, a_2|w(\cdot)) = L(a_1, a_2) \geq W(a_1, a_2|\overline{u}),$$

where the last inequality is strict when $a_2 < s(a_1|\overline{u})$. Thus, any L-IC$_2$ contract satisfies (P). Hence (P) is redundant. The argument extends to $a_2 = \underline{a}_2$, since the incentive constraint here is that $CE_{-V_2}(a_1, a_2|w(\cdot)) \geq L(a_1, a_2)$.

Proposition 3 *Assume A1–A5 hold, fix a_1, and assume that $s(a_1|\overline{u}) > \underline{a}_2$. Then, the participation constraint is redundant if the (a_1, a_2) action that is to be induced satisfies $a_2 \leq s(a_1|\overline{u})$.*

Laffont and Martimort (2002, Section 5.3) present a single-task model without private rewards. However, their utility function is not separable in income and effort. Although (P) is not redundant in their model, it may still be slack. In a standard separable model, an increase in the agent's base wage would necessitate an increase in performance pay in order to restore incentives. In Laffont and Martimort (2002), however, the agent is more risk averse when he works harder. The increase in base wage may then be more valuable if the agent works hard than if he does not. Thus, it may be possible to lower performance pay and maintain incentives at a reduced cost. This yields a more compressed wage schedule, which may also be to the agent's advantage and leave (P) slack.[6] Hence, there are some similarities between the models' conclusions. In the current model, a change in the contract changes a_2, which changes $V(w, a_2)$ and $c(a_1, a_2)$. This indirectly creates a type of non-separability between the contract and effort on the job, a_1.

Proposition 3 says that (P) is redundant when the action is held *fixed* in such a way that work-life balance is skewed away from "life" compared to the symmetric information benchmark. This does not mean that (P) is redundant or even slack

[6] The agent has only two actions available to him in Laffont and Martimort's (2002) model. Alvi (1997) justifies the first-order approach in a model that is fairly similar to Laffont and Martimort (2002) but with a continuous action. He does not discuss the participation constraint in any detail, but he does note that non-separability tends to make the contract flatter.

in the bigger problem where implementation costs are minimized over a_2. After all, the optimal a_2 could be large. This problem is addressed in the next section.

Corollary 1 *Assume A1–A5 hold, fix a_1, and assume that $s(a_1|\overline{u}) > \underline{a}_2$. Then, the agent must earn strictly more than reservation utility whenever his work-life balance is skewed further away from life than under symmetric information, i.e. whenever $a_2 < s(a_1|\overline{u})$. Conversely, if the agent only earns reservation utility, then his work-life balance must be skewed towards life, or $a_2 > s(a_1|\overline{u})$.*

Finally, $V(\cdot, a_2)$ and $-V_2(\cdot, a_2)$ are equally risk averse in the multiplicative model and (16) holds as an equality. This is why any action with $a_2 > s(a_1|\overline{u})$ cannot be implemented. However, outside the multiplicative model, actions for which $a_2 > s(a_1|\overline{u})$ may be implementable and may or may not leave (P) slack. It is impossible to determine exactly how large a_2 must be for (P) to come into play. Given how $W(a_1, a_2|\overline{u})$ explodes with a_2 it is natural that (P) binds eventually.

4 Optimal contracts and work-life balance

The fact that (P) can sometimes be ignored leads to the idea of examining a "reduced problem" where the only constraints are L-IC$_1$ and L-IC$_2$. The reduced problem predicts implementation costs that are increasing in a_2 when c_{12} is sufficiently high. Thus, there are cases where the optimal asymmetric information value of a_2 is below $s(a_1|\overline{u})$, in which case the reduced problem is indeed valid.

4.1 Implementation costs

Fix an action that is to be implemented. The principal now aims to minimize the cost of implementing this action. To this end, consider the *reduced problem*

$$\max_{w^R} - \int w^R(x_1) g_1(x_1|a_1) dx_1$$
$$st. \qquad EU_i(a_1, a_2|w^R(\cdot)) = 0, \quad i = 1, 2. \qquad \text{(L-IC)}$$

This is a reduced problem because it ignores (P) and implicitly assumes that L-IC is sufficient for incentive compatibility. Thus, solving the reduced problem must give a lower bound on implementation costs for *any* interior action. Let $E[w^R|a_1, a_2]$ denote this bound and $E[w|a_1, a_2]$ the true implementation costs.

It turns out that the contract that solves the reduced problem is regular and therefore incentive compatible. The contract must then also satisfy (P) if $a_2 \leq s(a_1|\overline{u})$, by Proposition 3. For such actions, the reduced problem thus correctly identifies the optimal contract. The argument holds even if $a_2 = \underline{a}_2$ because the cheapest way to satisfy the incentive constraint that $EU_i(a_1, a_2|w^R(\cdot)) \leq 0$ or $CE_{-V_2}(a_1, a_2|w^R(\cdot)) \geq L(a_1, a_2)$ is to make it bind, as in the reduced problem.

Theorem 1 *Assume that A1–A5 and P1 holds. For any action, the solution to the reduced problem is a regular contract that takes the form in (7), with $\lambda = 0$ and $\mu_1 > 0 > \mu_2$. Moreover, the optimal contract that induces a particular action (a_1, a_2) solves the reduced problem if $a_1 > \underline{a}_1$, $s(a_1|\overline{u}) > \underline{a}_2$, and $a_2 \leq s(a_1|\overline{u})$.*

Thus, holding a_1 fixed, the reduced problem characterizes the lowest possible implementation costs, or $E[w|a_1, a_2] = E[w^R|a_1, a_2]$, for any $a_2 \leq s(a_1|\overline{u})$. For higher a_2, the reduced problem describes a lower bound on implementation costs. The idea is to use $E[w^R|a_1, a_2]$ to proxy $E[w|a_1, a_2]$ in the first step of the solution procedure. The second step holds fixed a_1 and uses $E[w^R|a_1, a_2]$ to identify a candidate for an optimal level of a_2, with $a_2^R \in \arg\min_{a_2} E[w^R|a_1, a_2]$, to induce alongside a_1. If there is a solution with $a_2^R \leq s(a_1|\overline{u})$, then Theorem 1 applies and an optimal contract that induces a_1 has been identified.

Corollary 2 *Assume A1–A5 and P1 hold, fix a_1, and assume that $s(a_1|\overline{u}) > \underline{a}_2$. If $E[w^R|a_1, a_2]$ is minimized with respect to a_2 at any $a_2^R \leq s(a_1|\overline{u})$, then $E[w|a_1, a_2]$ is also minimized at a_2^R and implementation costs are $E[w^R|a_1, a_2^R]$.*

A weaker but still sufficient condition is that the contract from the reduced problem satisfies (P), which can always be checked after the reduced problem has been solved. However, this is automatic, and need not be checked, if $a_2^R \leq s(a_1|\overline{u})$. Section 5 outlines a general solution method that rigorously solves the problem regardless of what the second-best action is and whether or not (P) binds.

Leaning on Corollary 2, the next aim is to understand when a level of a_2 below $s(a_1|\overline{u})$ is optimal. The discussion of L-IC$_2$ in Section 3.1 might seem to suggest that low a_2 cannot be optimal as the job must be made very attractive in order to induce the agent to divest from life. However, with L-IC$_1$ necessitating variable pay, this is not the same as saying that the expected wage must be high. The job can be attractive if the wage is less risky. Now note that by disincentivizing life, it is precisely the case that flatter incentives are required to incentivize work. The question then is whether these flatter incentives on their own make the job attractive enough to disincentivize life. If so, the expected wage can be reduced to the principal's benefit. If not, the expected wage must be increased to discourage life, but then it is better to incentivize the agent to work harder at life instead. Thus, there is a trade-off when incentivizing lower a_2.

Another way of phrasing the trade-off is to note that lower a_2 makes it easier to induce a_1 and thus relaxes the L-IC$_1$ constraint. However, the agent's incentive to push a_2 higher must be kept in check and tightens the L-IC$_2$ constraint. The optimal level of a_2 weighs off these costs and benefits. Example 1 confirms that implementation costs may be increasing or decreasing in a_2 due to this trade-off.

It is possible to more precisely identify when one side of the trade-off dominates the other. Holding fixed the agent's reward function, the contract that solves the reduced problem relies on c_1 and c_2 but it is independent of any other features of the cost function, like c_{22} or c_{12}. By the Envelope Theorem, however, the latter affect how implementation costs change with a marginal change in a_2. It turns out that it is always possible to find a cost function that satisfies Assumption A4 and for which $E[w^R|a_1, a_2]$ is locally increasing in a_2 and another for which it is locally decreasing in a_2.

Proposition 4 *Fix the agent's reward function and fix an interior action* (a_1^*, a_2^*) *and* $(c_1(a_1^*, a_2^*), c_2(a_1^*, a_2^*))$. *Then, there exists cost functions that satisfy Assumption A4 and for which* $E[w^R|a_1, a_2]$ *is locally increasing (decreasing) in* a_2 *at* (a_1^*, a_2^*) *if* $c_{12}(a_1^*, a_2^*)$ *is large (small) relative to* $c_{22}(a_1^*, a_2^*) > 0$.

Proposition 4 implies that implementation costs are locally increasing in a_2 around e.g. $s(a_1|\overline{u})$ if c_{12} is large relative to c_{22}. A large value of c_{12} – or a large degree of substitutability between a_1 and a_2 – implies that it becomes much easier to satisfy the L-IC$_1$ constraint when a_2 is lowered. At the same time, a small value of c_{22} means that the L-IC$_2$ constraint is impacted less by a decrease in

a_2 and only becomes slightly harder to satisfy despite having lowered a_2. Thus, Corollary 2 is more likely to come into effect when c_{12} is large relative to c_{22}.[7]

A large value of c_{12} and a small value of c_{22} mean that the marginal costs of a_1 is impacted more than the marginal costs of a_2 when a_2 increases. Imagine, for instance, that a_2 is the number of hours the agent works a night shift on a tedious but otherwise undemanding assembly line for another employer. It is plausible that a significant side effect of an extra hour on the line is that it is much more difficult to concentrate on the more demanding primary job the next day.

Since $E[w^R|a_1, a_2]$ is independent of $\overline{u}$, so is the value (or values) of a_2 that minimizes it. In contrast, $s(a_1|\overline{u})$ is decreasing in $\overline{u}$. Hence, when $\overline{u}$ is very low, the agent is desperate for rewards and he works very hard at life under symmetric information. Thus, the condition in Corollary 2 is more likely to hold the lower $\overline{u}$ is. Once this condition is met, the optimal work-life balance is unaffected by further decreases in $\overline{u}$ under asymmetric but not under symmetric information. Thus, the two diverge. In other words, the lower $\overline{u}$ is, the more likely is it that the agent's work-life balance is skewed away from life and (P) is slack.

Finally, it is worthwhile to complete the circle and reconsider steepening or flattening the wage schedule, as in Section 2.2. Fix some $a_1 = a_1^* > \underline{a}_1$. Consider some $a_2^* \in \arg\min_{a_2} E[w|a_1^*, a_2]$, such that a_2^* is an optimal level of life to implement alongside a_1^*. Let $w^*(x_1)$ denote the contract that induces (a_1^*, a_2^*) and assume that $a_2^* > \underline{a}_2$ and that $a_2^* \leq s(a_1|\overline{u})$. Consider some alternative contract,

$$\widehat{w}_\beta(x_1) = (1-\beta)w^*(x_1) + \beta\widehat{w}(x_1),$$

where $\widehat{w}(x_1)$ also has the same expected wage as $w^*(x_1)$ in the event that the agent chooses $a_1 = a_1^*$. However, $\widehat{w}(x_1)$ and thus $\widehat{w}_\beta(x_1)$ may be steeper or flatter than $w^*(x_1)$. If $\beta = 0$, the contract $\widehat{w}_\beta(x_1)$ is simply the original contract $w^*(x_1)$. The idea is now to perturb $w^*(x_1)$ by marginally increasing β, starting from $\beta = 0$, and examining the resulting change in a_1.

Corollary 3 *Assume that* (a_1^*, a_2^*) *is interior with* $a_2^* \in \arg\min_{a_2} E[w|a_1^*, a_2]$ *and*

[7] Care should be taken when comparing Proposition 4 and Example 1. The former is a local comparative statics result that fixes an action but allows different cost functions to be compared. Example 1 fixes a specific cost function but allows global changes in the action.

$a_2^* \leq s(a_1^*|\overline{u})$. Then, there is no first-order effect on a_1 from a small mean-preserving perturbation of the optimal contract, $w^*(x_1)$.

The proof in the Appendix is instructive and ties together the optimal shape of the contract in (7) with Proposition 4 and the style of arguments in Proposition 2. Corollary 3 implies that the optimal contract is so finely tuned that a perturbation has no impact on effort on the job, although a_2 changes. The direct effect on work incentives that comes from the contract change is exactly nullified or balanced out by the indirect effect on work incentives that results from the induced change in a_2. This is just another way of saying that a_2^* is optimal; inducing a marginal change in a_2 has no first-order effect on the cost of inducing a_1^*. More generally, it is intuitive that there is no contract with the same expected wage that induces a higher a_1. Otherwise, the new contract could be shifted down, as in Proposition 1, to reincentivize a_1^* at a lower cost. However, this contradicts the assumption that the original contract is optimal. The use of Proposition 1 formally requires that the contract is regular but the intuition seems to be more general.

4.2 Work-life balance and flatter incentives

In Example 1, any contract that induces a low level of life features flatter incentives than the contract that would have maintained the symmetric-information level of work-life balance. At a broad intuitive level this result carries over to the general model, with the caveat that "weaker incentives" is a poorly defined term that is hard to formalize given that optimal contracts are typically non-linear.

However, it is unambiguously true that the agent's marginal cost of effort on the job, $c_1(a_1, a_2)$, is lower and his marginal utility of labor income, as measured by $V_1(w, a_2)$, is higher when a_2 is small. Hence, the contract must be made flatter in some average sense in order to restore incentives on the job when a smaller a_2 is induced. The Online Appendix presents one possible way of formalizing this.

4.3 Effort on the job

The logic that lead to Proposition 4 also makes it possible to examine how implementation costs vary with effort on the job, a_1.

Proposition 5 *Fix the agent's reward function and fix an interior action* (a_1^*, a_2^*) *and* $(c_1(a_1^*, a_2^*), c_2(a_1^*, a_2^*))$*. Then, there exist cost functions that satisfy Assumption A4 and for which* $E[w^R|a_1, a_2]$ *is locally increasing (decreasing) in* a_1 *at* (a_1^*, a_2^*) *if* $c_{11}(a_1^*, a_2^*) > 0$ *is large (small) relative to* $c_{12}(a_1^*, a_2^*)$.

There are two competing effects from incentivizing higher effort on the job. First, the marginal cost of effort on the job, c_1, changes, with c_{11} measuring the size of the change. Likewise, the marginal cost of effort in pursuit of private rewards, c_2, changes, with c_{12} capturing the speed of the change. When c_2 changes quickly (c_{12} is large), the agent who works harder on the job is much less inclined to work hard outside the office. Thus, L-IC$_2$ becomes substantially cheaper to satisfy. If c_1 changes slowly at the same time (c_{11} is small), then the cost of satisfying L-IC$_1$ only increases slightly. Then, the combined costs of L-IC$_1$ and L-IC$_2$ fall and it is cheaper to induce marginally higher effort on the job.

The fact that implementation costs may be locally decreasing in effort on the job is in marked contrast with the standard model. However, remember that a_2 typically adjusts alongside a_1 in the current model.

5 Discussion and extensions

This section briefly discusses some elements of, and extensions to, the basic model. The Online Appendix contains a more thorough discussion of assumptions and extensions. The model's relationships to the literatures on private investments and common agency are also reviewed there.

5.1 The benefit function

For conceptual simplicity, the principal's benefit function was assumed to depend only on the agent's effort on the job, a_1. However, this is immaterial to the reduced problem, which is concerned only with implementation costs but not at all with benefits. Thus, it is possible to allow the benefit function to depend on a_2 as well, in which case it can be written, with some abuse of notation, as $B(a_1, a_2)$. Step one of the three-step solution procedure is unaffected.

Depending on the application, $B(a_1, a_2)$ may be increasing or decreasing in a_2. For instance, in Holmström and Milgrom's (1991) example of the multi-tasking teacher, it is natural to assume that the benefit to the public is increasing in effort directed towards teaching either type of skill. In many other settings, however, $B(a_1, a_2)$ is likely to be decreasing in a_2. This might be the case if, for instance, the agent is moonlighting with a competitor to the principal. Alternatively, a_2 might describe a government official's inclination to engage in corrupt activities. Finally, Milgrom and Roberts (1988) describe many examples of "influence activities" internally in an organization. Here, an employee exerts effort to influence or bias the decisions of his superiors to his own gain.

Recall that (P) is slack if a small a_2 is induced and that this – as demonstrated by Example 1 and explained by Proposition 4 – may be optimal even if $B(a_1, a_2)$ is independent of a_2. If $B(a_1, a_2)$ is decreasing in a_2 then there is an additional reason to induce smaller a_2 and it becomes even more likely that (P) is slack.

5.2 Risk neutrality and limited liability

The agent has been assumed to be risk averse. It is well known that a moral hazard problem may also exist in environments where the agent is risk neutral but is protected by a limited liability constraint. The easiest setting in which to think about such a problem is when private rewards are monetary as well and $v(w, x_2) = w + x_2$. This is fundamentally different from the model analyzed above because now $v_{12} = 0$ rather than $v_{12} < 0$. This lack of interdependency between rewards means that a_1 and a_2 are related only through the cost function, $c(a_1, a_2)$. As explained more thoroughly in Example 2 of the Online Appendix, this implies that for any a_1 that is to be implemented, there is exactly one level of a_2 that can be induced alongside it. Consequently, the agent's action is not truly multi-dimensional. In sum, with a risk neutral agent – or more precisely when $v_{12}(w, x_2) = 0$ – the model essentially reduces to a standard one-dimensional principal-agent model, whether limited liability is a factor or not. Incidentally, the LEN model should be seen in this light, since in that model the agent's certainty equivalent is additively separable in rewards.

5.3 Bounded actions

Possible complications arise if a_2 is bounded above by some $\overline{a}_2 < \infty$. The starkest way to see this is to assume that $\overline{u}$ is very low. Then, (P) must bind at $\overline{a}_2$. Otherwise, the contract can be shifted down, as in Proposition 1, until (P) binds. This works because it is impossible to further increase a_2. Formally, incentive compatibility at $(a_1, \overline{a}_2)$ requires $EU_2 \geq 0$, but this only places an upper bound on CE_{-V_2}. Moreover, when $\overline{u}$ is very low, the reduced problem is valid for all $a_2 < \overline{a}_2$ and so (P) is slack whenever $a_2 < \overline{a}_2$ is implemented. Thus, there is a discontinuity at $a_2 = \overline{a}_2$, and implementation costs drop here. A further reduction in $\overline{u}$ does not effect the reduced problem or the implementation costs of $a_2 < \overline{a}_2$. However, it lowers the cost of implementing $\overline{a}_2$, since these costs are determined by a binding (P). In other words, when $\overline{u}$ is very low, the second-best action involves $a_2 = \overline{a}_2$, (P) binds, and $EU_2 \geq 0$ is slack. Consequently, the contract looks exactly the same as in a standard model.

5.4 The First-Order Approach

The reduced problem describes how to optimally implement any given action with $a_2 \leq s(a_1|\overline{u})$. However, the second-best action may in some cases involve higher a_2 and the reduced problem may then be unable to identify the optimal contract. This problem is addressed here, where a slightly different solution method is developed. The principal's benefit function is allowed to be non-increasing in a_2.

Assumption P2 (The benefit function): The principal is risk neutral and never benefits from higher a_2, i.e. $B_2(a_1, a_2) \leq 0$ for all (a_1, a_2).

With Assumption P2 in hand, the so-called first-order approach can be justified. The first-order approach maximizes the principal's net payoff subject only to (P) and L-IC, as was discussed before deriving (7). The proof of the validity of the first-order approach utilizes and extends arguments in Rogerson (1985).[8]

Theorem 2 *Assume that A1–A5 and P2 hold and that any second-best action* (a_1, a_2) *is interior. Then, the first-order approach is valid. The optimal contract*

[8] For more details and discussion see the working paper version, Kirkegaard (2016), where the main focus is on justifying the first-order approach.

is regular and takes the form in (7), with $\mu_1 > 0 > \mu_2$.

Proof. See the Appendix. ■

6 Conclusion

This paper extends the canonical principal-agent model to allow the agent to pursue private, stochastic, and possibly non-monetary rewards. Conceptually, this way of "unpacking" leisure recognizes that rewards earned while not on the job are also endogenous. Hence, the principal manipulates not only the agent's effort on the job but also his "work-life balance" through the contract design.

The model's non-separability between rewards from labor income and other sources is central to the paper's economic insights. For instance, it makes the participation constraint slack for certain actions. Likewise, the tendency to substitute toward life and away from work complicates the incentive effects of shifting or steepening the contract.

Likewise, at the technical level, it is also this non-separability that represents the main challenge. Thus, the paper's technical contribution is to propose a solution method to deal with private rewards without assuming separability. Recall that separability is implicit in the LEN model, for instance. Thus, the paper's results demonstrate that non-separability have important economic implications.

In Englmaier and Leider (2012), the agent is awarded high utility to trigger feelings of reciprocity. A change in government policy that increases the value of the outside option makes the principal seem relatively less generous. The agent then becomes less intrinsically motivated and the principal will typically want to change the contract.[9] In standard models, contract design is also very sensitive to the outside option since the participation constraint binds. In the current model, a small change in reservation utility does not impact the problem when the participation constraint is redundant. Hence, there is no need to change the contract. Thus, the contract is less sensitive to changes in the outside option.

[9] A similar effect exists in the efficiency-wage model of Shapiro and Stiglitz (1984). There, higher unemployment benefits decrease the penalty to being fired for shirking. To restore incentives, firms are forced to increase wages, which in turns leads to increased unemployment.

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Appendix

Proof of Lemma 1. Integration by parts with respect to x_2 yields

$$EU(a_1, a_2) = \int \left(v(w(x_1), \overline{x}_2) - \int v_2(w(x_1), x_2) G^2(x_2|a_2) dx_2 \right) g^1(x_1|a_1) dx_1 - c(a_1, a_2). \tag{17}$$

It is well known that Assumption A2 (MLRP) implies that $G^i_{a_i}(x_i|a_i) < 0$ for all interior x_i. Moreover, Assumptions A2 and A3 together imply that $G^i_{a_i a_i}(x_i|a_i) > 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$, $i = 1, 2$. To see this, note that A3 (LOCC) necessitates that $G^i_{a_i a_i} \geq 0$ and $G^1 G^2 G^1_{a_1 a_1} G^2_{a_2 a_2} - \left(G^1_{a_1} G^2_{a_2}\right)^2 \geq 0$. At any interior (x_1, x_2), the last term is strictly positive, by A2 (MLRP). Thus, $G^1_{a_1 a_1} > 0$ and $G^2_{a_2 a_2} > 0$ are necessary. Since $v_2 > 0$, the first term in (17) is therefore strictly concave in a_2. The second term is weakly concave in a_2 by Assumption A4. Thus, $EU_{22} < 0$.

Assuming the contract is regular, another round of integrating by parts, this time with respect to x_1, yields

$$\begin{aligned} EU(a_1, a_2|w(\cdot)) \quad = \quad & -\int v_1(w(x_1), \overline{x}_2) w'(x_1) G^1(x_1|a_1) dx_1 - \int v_2(w(\overline{x}_1), x_2) G^2(x_2|a_2) dx_2 \\ & + \int\int v_{12}(w(x_1), x_2) w'(x_1) G^1(x_1|a_1) G^2(x_2|a_2) dx_1 dx_2 \\ & + v(w(\overline{x}_1), \overline{x}_2) - c(a_1, a_2). \end{aligned} \tag{18}$$

Thus,

$$EU_{12}(a_1, a_2|w(\cdot)) = \int\int v_{12}(w(x_1), x_2) w'(x_1) G^1_{a_1}(x_1|a_1) G^2_{a_2}(x_2|a_2) dx_1 dx_2 - c_{12}(a_1, a_2).$$

Since $G^i_{a_i}(x_i|a_i) < 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$, $i = 1, 2$, the last two parts of Assumption A4, $v_{12} < 0$ and $c_{12} \geq 0$, imply that $EU_{12}(a_1, a_2|w(\cdot)) < 0$ for all regular contracts. A similar argument proves that $EU_{11}(a_1, a_2|w(\cdot)) < 0$ if the contract is regular.

Since $v_1, v_2 > 0 > v_{12}$ and $G^1(x_1|a_1)$, $G^2(x_2|a_2)$, $G^1(x_1|a_1)G^2(x_2|a_2)$, and $c(a_1, a_2)$ are all convex in (a_1, a_2), it follows from (18) that $EU(a_1, a_2|w(\cdot))$ is concave because it is the sum of concave functions. To prove that $EU_{11}EU_{22} - EU^2_{12} > 0$ when $w(x_1)$ is regular, let $P(a_1, a_2)$ denote the first line in (18) and let $Q(a_1, a_2)$ denote the remainder, such that $EU = P + Q$. Note that $P_{11}, P_{22} < 0$

but $P_{12} = 0$. Similarly, $Q_{11}, Q_{22} < 0$ and by concavity $Q_{11}Q_{12} - Q_{12}^2 \geq 0$. Now,

$$
\begin{aligned}
EU_{11}EU_{22} - EU_{12}^2 &= \left[P_{11}P_{22} - P_{12}^2\right] + \left[Q_{11}Q_{22} - Q_{12}^2\right] + \left[P_{11}Q_{22} + P_{22}Q_{11} - 2P_{12}Q_{12}\right] \\
&= \left[P_{11}P_{22}\right] + \left[Q_{11}Q_{22} - Q_{12}^2\right] + \left[P_{11}Q_{22} + P_{22}Q_{11}\right] > 0,
\end{aligned}
$$

since the first and third terms are strictly positive and the second term is non-negative. This implies strict concavity. ■

Proof of Proposition 1. For brevity, write EU for $EU(a_1, a_2|w(\cdot))$. Since $w(x_1)$ is regular, Lemma 1 implies that $EU_{11} < 0$, $EU_{22} < 0$, and $EU_{12} < 0$. Moreover, $EU_{11}EU_{22} - EU_{12}^2 > 0$, or

$$
\frac{-EU_{12}}{EU_{22}} > \frac{-EU_{11}}{EU_{12}}. \tag{19}
$$

Given the agent's problem is concave, the optimal action is determined by the first order conditions $EU_1 = 0$ and $EU_2 = 0$. In (a_1, a_2) space, the curves along which $EU_1 = 0$ and $EU_2 = 0$ have slope

$$
\frac{da_2}{da_1}_{|EU_1=0} = \frac{-EU_{11}}{EU_{12}} < 0 \text{ and } \frac{da_2}{da_1}_{|EU_2=0} = \frac{-EU_{12}}{EU_{22}} < 0.
$$

The optimal interior action (a_1^*, a_2^*) is found where these two curves intersect. By (19) the curve where $EU_1 = 0$ crosses the curve where $EU_2 = 0$ exactly once, from above. See Figure 3.

Similarly, write EU^{ε} for $EU(a_1, a_2|\widehat{w}_{\varepsilon}(\cdot))$. Note that $\widehat{w}_{\varepsilon}(x_1)$ is also a regular contract. Next, note that by design of $\widehat{w}_{\varepsilon}(x_1)$, $EU_1^{\varepsilon} = 0$ at (a_1^*, a_2^*). That is, both the $EU_1 = 0$ curve and the $EU_1^{\varepsilon} = 0$ curve go through the point (a_1^*, a_2^*). However, by (17) in Lemma 1,

$$
\begin{aligned}
EU_2(a_1, a_2|w(\cdot)) &= -\int \left(\int v_2(w(x_1), x_2) G_{a_2}^2(x_2|a_2) dx_2 \right) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) \\
&< -\int \left(\int v_2(\widehat{w}_{\varepsilon}(x_1), x_2) G_{a_2}^2(x_2|a_2) dx_2 \right) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) \\
&= EU_2(a_1, a_2|\widehat{w}_{\varepsilon}(\cdot)),
\end{aligned}
$$

or $EU_2 < EU_2^{\varepsilon}$ for all (a_1, a_2). The inequality follows from $v_{12} < 0$, $\widehat{w}_{\varepsilon}(x_1) <$

$w(x_1)$, and $G^2_{a_2} < 0$. Then, $EU^{\varepsilon}_{22} < 0$ implies that the curve where $EU^{\varepsilon}_2 = 0$ lies above the curve where $EU_2 = 0$. This rules out that $a_1^{'} = a_1^*$. See Figure 3.

Imagine that $a_1^{'} > a_1^*$. Then, since $EU^{\varepsilon}_1 = 0$ crosses the curve where $EU^{\varepsilon}_2 = 0$ from above at (a_1', a_2'), the $EU^{\varepsilon}_1 = 0$ curve must lie above the $EU^{\varepsilon}_2 = 0$ curve for any $a_1 < a_1'$. Thus, $EU^{\varepsilon}_1(a_1^*, a_2^*) > 0$, which is a contradiction. Thus, $a_1^{'} < a_1^*$. As the $EU^{\varepsilon}_2 = 0$ curve lies above the $EU_2 = 0$ curve, it then follows that $a_2' > a_2^*$. ■

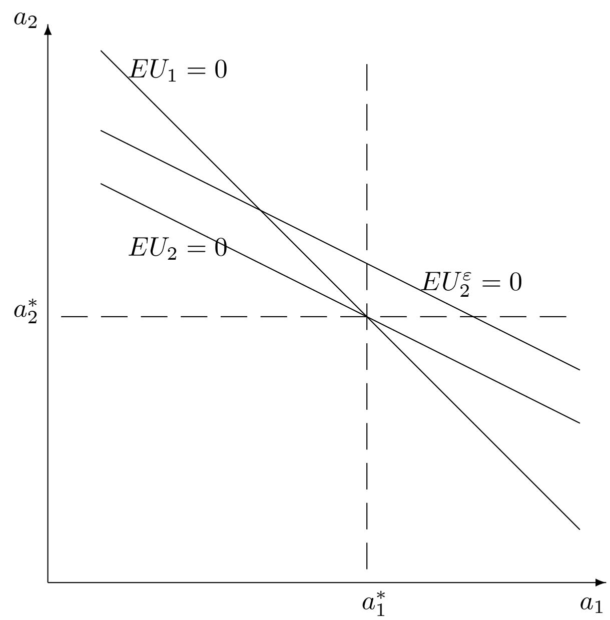

Figure 3: Solving the agent's problem.

Proof of Proposition 2. The same logic as in Proposition 1 is used. First,

$$EU_1(a_1^*, a_2^*|\widehat{w}_\alpha(\cdot)) - EU_1(a_1^*, a_2^*|w(\cdot)) = \int \left[V(\widehat{w}_\alpha(x_1), a_2^*) - V(w(x_1), a_2^*)\right] g^1_{a_1}(x_1|a_1^*)dx_1 > 0$$

since both terms under the integral have the same sign. Thus,

$$EU_1(a_1^*, a_2^*|\widehat{w}_\alpha(\cdot)) > EU_1(a_1^*, a_2^*|w(\cdot)) = 0.$$

Similarly,

$$\begin{aligned} EU_2(a_1^*, a_2^* | \widehat{w}_\alpha(\cdot)) &= \int V_2(\widehat{w}_\alpha(x_1), a_2^*) g^1(x_1|a_1^*) dx_1 - c_2(a_1^*, a_2^*) \\ &> \int V_2(w(x_1), a_2^*) g^1(x_1|a_1^*) dx_1 - c_2(a_1^*, a_2^*) \\ &= EU_2(a_1^*, a_2^* | w(\cdot)) = 0, \end{aligned}$$

where the inequality comes from the assumptions that V_2 is convex in w and that the distribution of wages under $\widehat{w}_\alpha(\cdot)$ is a mean-preserving spread over wages under $w(x_1)$. The curvature of V_2 in determining incentives is important throughout the paper and is discussed in more detail later.

To continue, let EU^α denote $EU(a_1, a_2|\widehat{w}_\alpha(\cdot))$. It has been shown that $EU_i^\alpha > EU_i$ at (a_1^*, a_2^*), $i = 1, 2$. In words, the direct effect of the contract change is to create incentives to increase both actions. However, as a_2 increases, an incentive to lower a_1 is created, which leads to a trade-off. To capture this formally, start by keeping fixed a_1 at a_1^*. As a_2 increases, both EU_1^α and EU_2^α strictly decreases since $EU_{12}^\alpha < 0$ and $EU_{22}^\alpha < 0$ by Lemma 1. If c_{12} is large enough compared to c_{22}, then EU_1^α decreases faster than EU_2^α in a_2. If this effect is large enough, then EU_1^α reaches zero before EU_2^α does. Thus, there is some $a_2'' > a_2^*$ such that $EU_1^\alpha = 0$ and $EU_2^\alpha > 0$ at (a_1^*, a_2''). The level curve along which $EU_2^\alpha = 0$ lies above this point. Since both level curves, $EU_1^\alpha = 0$ and $EU_2^\alpha = 0$, are downwards sloping, their intersection is to the north-west of (a_1^*, a_2''). This intersection, (a_1', a_2'), describes the new equilibrium action and satisfies $a_1' < a_1^*$ and $a_2' > a_2'' > a_2^*$. $\blacksquare$

Proof of Lemma 2. First,

$$V_1(w, a_2) = \int v_1(w, x_2) g^2(x_2|a_2) dx_2.$$

Assumption A4 implies that $V(w, a_2)$ is strictly increasing and strictly concave in w, or $V_1(w, a_2) > 0 > V_{11}(w, a_2)$, just like a standard utility function. Moreover, A2 (MLRP) and A4 together imply that

$$V_{12}(w, a_2) = \int v_1(w, x_2) g_{a_2}^2(x_2|a_2) dx_2 < 0.$$

The reason is that $v_1(w, x_2)$ is strictly decreasing in x_2 (A4) and that an increase in a_2 makes higher x_2 more likely (A2). Similarly, Assumption A4 together with A2 and A3 (LOCC) imply that $V_2(w, a_2) > 0$ and $V_{22}(w, a_2) < 0$, respectively. This proves the first two parts of the lemma.

For the third part, note that A2 (MLRP) is equivalent to the requirement that $g^i(x_i|a_i)$ is log-supermodular in (x_i, a_i), $i = 1, 2$. Then, the term under the integration sign in $V_1(w, a_2)$ is, by A2 (MLRP) and A5 (Log-supermodularity), log-supermodular in *all* (three) arguments, (w, x_2, a_2). In this case, log-supermodularity is preserved under integration so $V_1(w, a_2)$ is log-supermodular in its (two) arguments (w, a_2); see e.g. Athey (2002, p. 193). This proves the third part of the lemma.

For the fourth part, $-V_2(w, a_2)$ is increasing in w by part 2, which means that $-V_{12}(w, a_2) > 0$. Similarly, the term $\frac{-V_{12}}{V_1}$ in part 3 is weakly decreasing in w. This is precisely equivalent to $-V_2$ having a larger coefficient of absolute risk aversion than V, or $-V_2$ being a concave transformation of V. Formally, this can be seen by carrying out the differentiation in (15). Since V is risk averse, this necessitates that $-V_2$ is risk averse too, or $-V_{112} < 0$. ■

Proof of Lemma 3. Given $\mu_2 \leq 0$, $V_{11} < 0$ and (15) imply that the right hand side of (7) is strictly increasing in w (the derivative is strictly positive). Thus, for each x_1 there is at most one solution to (7), $w(x_1)$. Differentiability now follows from the differentiability of all the components in (7) and the fact that the right hand side is strictly increasing in w. If $\mu_1 > 0$, A2 (MLRP) implies that the left hand side is strictly increasing in x_1. Hence, the contract is regular. ■

Proof of Proposition 3. The proof is in the text. ■

Proof of Corollary 1. The corollary follows straight away from Proposition 3. ■

Proof of Theorem 1. The reduced problem produces a contract of the form in (7), but with $\lambda = 0$. Thus, there exists a value of x_1 for which the left hand side is zero. Any solution to (7) then requires $\mu_2 < 0$. Hence, the right hand side is strictly increasing in w, by A4 and A5. If $\mu_1 \leq 0$, the contract is weakly decreasing, by A2. In this case, $EU_1 < 0$ which violates one of the constraints of the reduced problem. Hence, $\mu_1 > 0$. The contract is regular by Lemma 3.

This completes the first part of the proof. For the second part, regularity and Lemma 1 implies that the contract is incentive compatible. By the assumption that $a_2 \leq s(a_1|\overline{u})$, (P) is satisfied. Hence, the contract is feasible. Thus, it solves the principal's problem, given the action. ■

Proof of Corollary 2. The proof is in the text. ■

Proof of Proposition 4. The objective function in the reduced problem is $-E[w^R|a_1, a_2]$. By the Envelope Theorem,

$$\frac{\partial E[w^R|a_1,a_2]}{\partial a_1} = \int w^R(x_1) g^1_{a_1}(x_1|a_1)dx_1 - \mu_1 EU_{11} - \mu_2 EU_{12} \tag{20}$$

$$\frac{\partial E[w^R|a_1,a_2]}{\partial a_2} = -\mu_1 EU_{12} - \mu_2 EU_{22}. \tag{21}$$

Given (a_1^*, a_2^*), $\mu_1 > 0$ and $\mu_2 < 0$ depend, through L-IC, only on $c_1(a_1^*, a_2^*)$ and $c_2(a_1^*, a_2^*)$ but not on $c_{11}(a_1^*, a_2^*)$, $c_{22}(a_1^*, a_2^*)$, or $c_{12}(a_1^*, a_2^*)$. However, EU_{ij} clearly depends on c_{ij}. Thus, these terms in (20) and (21) can be manipulated by changing (c_{11}, c_{22}, c_{12}). However, Assumption A4 imposes the restrictions that $c_{11}, c_{22}, c_{12} \geq 0$ and $c_{11}c_{22} - c_{12}^2 \geq 0$. The two terms in (21) are independent of c_{11}. The first term is increasing and linear in c_{12} while the second term in decreasing and linear in c_{22}. Thus, (21) is positive (negative) when c_{22} is small (large) compared to c_{12}. Given $c_{22} > 0$, it is always possible to find a $c_{11} > 0$ for which A4 is satisfied. ■

Proof of Corollary 3. Given the contract $\widehat{w}_\beta(x_1)$, let $(a_1(\beta), a_2(\beta))$ denote the agent's action, ignoring (P), with $\widehat{w}_0(x_1) = w^*(x_1)$ and $a_i(0) = a_i^*$, $i = 1, 2$. L-IC$_1$ is

$$\int V(\widehat{w}_\beta(x_1), a_2(\beta)) g^1_{a_1}(x_1|a_1(\beta))dx_1 - c_1(a_1(\beta), a_2(\beta)) = 0.$$

Differentiating with respect to β and evaluating at $\beta = 0$ yields

$$k_1 + EU_{11}(a_1^*, a_2^*|w^*(\cdot))a_1'(0) + EU_{12}(a_1^*, a_2^*|w^*(\cdot))a_2'(0) = 0, \tag{22}$$

where

$$k_1 = \int V_1(w^*(x_1), a_2^*)\left(\widehat{w}(x_1) - w^*(x_1)\right) g^1_{a_1}(x_1|a_1^*)dx_1.$$

Similarly, from L-IC$_2$,

$$k_2 + EU_{12}(a_1^*, a_2^*|w^*(\cdot))a_1'(0) + EU_{22}(a_1^*, a_2^*|w^*(\cdot))a_2'(0) = 0, \tag{23}$$

where

$$k_2 = \int V_{12}(w^*(x_1), a_2^*)\left(\widehat{w}(x_1) - w^*(x_1)\right) g^1(x_1|a_1^*)dx_1.$$

Solving (22) and (23) yields

$$a_1'(0) = \frac{k_2 EU_{12}(a_1^*, a_2^*|w^*(\cdot)) - k_1 EU_{22}(a_1^*, a_2^*|w^*(\cdot))}{EU_{11}(a_1^*, a_2^*|w^*(\cdot))EU_{22}(a_1^*, a_2^*|w^*(\cdot)) - EU_{12}(a_1^*, a_2^*|w^*(\cdot))^2}.$$

The aim is to show that $a_1'(0) = 0$. Recall that EU_{12} and EU_{22} are strictly negative and that the denominator is strictly positive. Thus, these factors are all non-zero.

To proceed, note that

$$\begin{aligned}\mu_1 k_1 + \mu_2 k_2 = \int &\left(\mu_1 V_1(w^*(x_1), a_2^*)l_{a_1}^1(x_1|a_1^*) + \mu_2 V_{12}(w^*(x_1), a_2^*)\right) \\ &\times \left(\widehat{w}(x_1) - w^*(x_1)\right) g^1(x_1|a_1^*)dx_1\end{aligned}$$

The first term under the integral is exactly 1 for all x_1. This follows from (7) and the fact that $\lambda = 0$ since $a_2^* \leq s(a_1^*|\overline{u})$. Then,

$$\mu_1 k_1 + \mu_2 k_2 = \int \left(\widehat{w}(x_1) - w^*(x_1)\right) g^1(x_1|a_1^*)dx_1,$$

which is zero by the assumption that $\widehat{w}(x_1)$ has the same expected value as $w^*(x_1)$ given $a_1 = a_1^*$. It follows that $\mu_1 k_1 = -\mu_2 k_2$. Since $\mu_1 > 0 > \mu_2$, either both k_1 and k_2 are zero, or neither of them are. If both are zero, then $a_1'(0) = 0$ and the proof is done. Otherwise, $\frac{k_2}{k_1} = \frac{-\mu_1}{\mu_2}$ and $a_1'(0)$ is proportional to

$$\begin{aligned}a_1'(0) &\propto k_1 EU_{12}(a_1^*, a_2^*|w^*(\cdot))\left(\frac{k_2}{k_1} - \frac{EU_{22}(a_1^*, a_2^*|w^*(\cdot))}{EU_{12}(a_1^*, a_2^*|w^*(\cdot))}\right) \\ &= k_1 EU_{12}(a_1^*, a_2^*|w^*(\cdot))\left(\frac{-\mu_1}{\mu_2} - \frac{EU_{22}(a_1^*, a_2^*|w^*(\cdot))}{EU_{12}(a_1^*, a_2^*|w^*(\cdot))}\right)\end{aligned}$$

Similarly, from the assumption that a_2^* is interior and optimal given a_1^*, it must hold that $\frac{\partial E[w^R|a_1^*,a_2^*]}{\partial a_2} = 0$. From (21) in the proof of Proposition 4, this implies that $\mu_1 EU_{12} = -\mu_2 EU_{22}$, or $\frac{-\mu_1}{\mu_2} = \frac{EU_{22}}{EU_{12}}$, which then again means that $a_1'(0) = 0$. This completes the proof. ■

Proof of Proposition 5. The proof follows from (20) in the proof of Proposition 4. In (20), the first two terms are positive while the third term is negative. All three terms are independent of c_{22}. Holding fixed c_{12} and c_{22}, the second term comes to dominate as $c_{11} > 0$ explodes. Thus, there is a cost function for which (20) is strictly positive. On the other hand, fix $c_{11} > 0$ and let $c_{12} > 0$ and $c_{22} > 0$ explode in such a way that the convexity condition $c_{11}c_{22} - c_{12}^2 \geq 0$ remains satisfied. Then, the last term dominates and (20) is strictly negative. ■

Proof of Theorem 2. Consider the following *relaxed problem*, so named because the incentive compatibility constraint in the original or "unrelaxed" problem has been weakened,

$$\max_{a_1,a_2,w} B(a_1,a_2) - \int w(x_1) g^1(x_1|a_1) dx_1$$

$$st. \quad EU(a_1,a_2) \geq \overline{u} \tag{P}$$

$$EU_i(a_1,a_2) = 0, \quad i = 1,2. \tag{L-IC}$$

The first-order approach is said to be valid if the solution to the relaxed problem also solves the original or unrelaxed problem and thus identifies the second-best. As in Rogerson (1985), a *doubly-relaxed* problem is utilized. In Rogerson's one-task model, the relaxed incentive compatibility constraint, $EU_1 = 0$, is replaced with the even weaker constraint that $EU_1 \geq 0$. In the current multi-task model, the appropriate doubly-relaxed problem assumes that

$$EU_1(a_1,a_2) \geq 0 \text{ and } EU_2(a_1,a_2) \leq 0.$$

Rogerson (1985) uses the doubly-relaxed problem to deal with the additional nonlinearities that arise from having a risk averse principal. Here, it is used instead to deal with nonlinearities from the additional incentive constraints.

Conveniently, $\mu_1 \geq 0 \geq \mu_2$ must hold in the doubly-relaxed problem. Moreover, any solution to the doubly-relaxed problem must take the form in (7), with $\mu_1 \geq 0 \geq \mu_2$. However, wages are constant if $\mu_1 = 0$. Then, $EU_1 = -c_1 < 0$, which violates the doubly-relaxed constraints. Hence, $\mu_1 > 0$ and so $EU_1 = 0$. By Lemma 3, any solution involves a regular contract. By Lemma 1, the agent's problem is concave. The contract is then incentive compatible if $EU_1 = EU_2 = 0$ at the intended action, which holds if $\mu_1 > 0 > \mu_2$.

Thus, the next step establishes that $\mu_2 < 0$. If an interior a_2 is optimal in the doubly-relaxed problem then the principal's first-order condition

$$B_2 + \lambda EU_2 + \mu_1 EU_{12} + \mu_2 EU_{22} = 0, \tag{24}$$

must hold. By Assumption P2, $B_2 \leq 0$. By Lemma 1, it holds that $EU_{12} < 0$ given the contract is regular. Since $\lambda EU_2 \leq 0$, the first three terms in (24) are thus strictly negative. As $EU_{22} < 0$, it is therefore necessary that $\mu_2 < 0$. Hence, $EU_1 = EU_2 = 0$.

By assumption, any second-best action is interior. Thus, any solution to the unrelaxed problem must satisfy $EU_1 = EU_2 = 0$, which implies that it is feasible in the doubly-relaxed problem. However, it has just been shown that any interior solution to the doubly-relaxed problem is feasible in the unrelaxed problem. Hence, the solutions to the unrelaxed and doubly-relaxed problems coincide. Finally, the set of feasible contracts is obviously larger in the doubly-relaxed problem than in the relaxed problem. Then, as the solution to the doubly-relaxed problem involves an interior action, the solution is also feasible in the relaxed problem, which must then identify the exact same solution. This completes the proof.

To clarify, note that the first-order approach justified here does not make it possible to derive implementation costs for actions other than the second-best (contrary to Theorem 1). Thus, it cannot be used as a means to extend the comparative statics in Proposition 4 to $a_2 > s(a_1|\overline{u})$ where the reduced problem may not be valid. ■

Online Appendix

The first section of this appendix is devoted to discussing the assumptions of the model. The assumptions are interpreted and in some cases justified more carefully. The relationship to some existing literature is spelled out in more detail and possible relaxations are discussed as well.

The second section describes the details of Example 1 from the main paper, using the multiplicative model with square-root utility. It includes a closed-form solution of the optimal contract that induces any given interior action as well as an explicit characterization of implementation costs. The third section argues more formally that incentives are flatter when smaller a_2 are induced.

The fourth section presents a reinterpretation of the reduced problem in which the agent is intrinsically motivated to work hard on the job. The fifth and final section examines the model's link to the literature on common agency.

A Assumptions

This section discusses Assumptions A1–A5 in more detail. An example is given in which A4 does not hold. This example clarifies how the model differs from, and is richer than, the Linear-Exponential-Normal model.

A.1 Assumptions A1–A2

Assumption A1 (independence) assumes that the signal x_1 and the private reward x_2 are independent. For example, there is little reason to think that job performance and the mastery of a hobby are correlated. In other settings, such as when the agent is moonlighting in the same industry, the independence assumption is harder to justify. However, the assumption may have some behavioral justification even in such cases. In particular, there is a growing literature on the prevalence and consequences of *correlation neglect*. See e.g. Levy and Razin (2015) and the references therein. In the current context, correlation neglect arises if the principal and the agent know the marginal distributions, but ignore any correlation between the random variables in the joint distribution.

There are at least two technical problems related to relaxing the independence assumption. One is to establish a counterpart to Lemma 1 for "well-behaved" contracts. Moreover, (7) may no longer apply. Thus, it also becomes harder to verify whether the contract is "well-behaved" in the first place. In short, A1 captures the main price of allowing the rewards function to be non-separable.

Assumption A2 (MLRP) ensures both that (i) the contract is regular and that (ii) a first order stochastic dominance property holds, i.e. that $G^i_{a_i}(x_i|a_i) < 0$ for $x_i \in (\underline{x}_i, \overline{x}_i)$. The assumption that $g^1(x_1|a_1)$ is log-supermodular can be replaced with the assumption that the first order stochastic dominance property holds and that there is an exogenous restriction that the contract must be non-decreasing in x_1. Such a restriction arises if the agent can sabotage the signal after it has been realized but before the principal observes it.

The assumption that $g^2(x_2|a_2)$ is log-supermodular plays a role in the aggregation result in Lemma 2. It can be replaced by the first order stochastic dominance property and the more direct assumption that (15) holds. Note that (15) is automatic in the multiplicative model.

A.2 Assumption A3

Assumption A3 (LOCC) is a technically motivated assumption that is instrumental in justifying the solution method. It is a direct extension of Rogerson's (1985) convexity assumption (CDFC). Recall that Rogerson assumes that there is a single signal and a single task. Conlon (2009) presents justifications of the first-order approach (FOA) that permit multiple signals but a single task. Kirkegaard (2017) allows multiple tasks, under the assumption that A1 holds. However, private rewards are ruled out and all signals are contractible. In the previous version of the current paper, Kirkegaard (2016), justifications of the FOA with private rewards are given that are in the spirit of Jewitt's (1988) single-task justifications.

A sufficient condition for LOCC is that G^1 and G^2 are both log-convex. The product of log-convex functions is itself log-convex, and therefore necessarily convex. Alternatively, fix some G^1 that is strictly convex in a_1, but not necessarily log-convex. Then, there is always some "sufficiently convex" G^2 function that ensures that Assumption A3 is satisfied. For example, a non-negative function

$h(z)$ is said to be ρ-convex if $h(z)^\rho/\rho$ is convex, or $h''(z)h(z)/h'(z)^2 \geq 1-\rho$ for all z. Thus, a ρ-convex function is log-convex if and only if $\rho \leq 0$ (and convex if and only if $\rho \leq 1$). If $G^2(x_2|a_2)$ satisfies Assumption A2 and is ρ-convex in a_2 (for all x_2) for some small enough ρ (i.e. ρ is negative, but numerically large), then Assumption A3 is satisfied. To see this, note first that the convexity assumption in A3 necessitates that $G^1 G^1_{a_1 a_1}(G^2 G^2_{a_2 a_2}/\left(G^2_{a_2}\right)^2) - \left(G^1_{a_1}\right)^2 \geq 0$ for interior (x_1, x_2). By ρ-convexity, the left hand side is greater than $G^1 G^1_{a_1 a_1}(1-\rho) - \left(G^1_{a_1}\right)^2 \geq 0$. Hence, the inequality is satisfied if ρ is small enough. To reiterate, as long as G^1 satisfies a strict version of CDFC there are G^2 functions that will permit the FOA to be justified even when allowing for private rewards.

There are some similarities between the current model of private rewards and the literature on hidden savings. Ábrahám et al. (2011) consider a situation where the agent works for the principal while simultaneously privately investing in a risk-free asset. There is thus no uncertainty concerning the return to the non-contractible action. Hence, performance on the job, x_1, is the only source of uncertainty. Ábrahám et al. (2011) justify the FOA by assuming that the distribution of x_1 is log-convex in effort on the job, a_1, and that the agent has decreasing absolute risk aversion. Assumptions A3 (LOCC) and A5 (log-supermodularity) in the current paper can be seen as extensions that allow returns that are both stochastic and potentially non-monetary.

More specifically, let a_2 denote the dollar amount that the agent saves. Savings has a risk-free rate of return of r. Letting $U(\cdot)$ denote the Bernoulli utility function over total wealth, the agent's utility upon earning $w(x_1)$ on the job and ra_2 from savings is $U(w(x_1)+ra_2)$. Given action (a_1, a_2), integration by parts yields expected utility from rewards of

$$\int U(w(x_1)+ra_2) g^1(x_1|a_1) dx_1 = U(w(\overline{x}_1)+ra_2) - \int U'(w(x_1)+ra_2) w'(x_1) G^1(x_1|a_1) dx_1. \tag{25}$$

The first term is concave in (a_1, a_2), given the agent is risk averse. Next, note that decreasing absolute risk aversion in total income is equivalent to log-convexity of $U'(\cdot)$. First, since $U'(\cdot)$ is log-convex in w, the right hand side of the counterpart to (7) is then well-behaved. Second, $U'(\cdot)$ is log-convex in a_2. Then, assuming G^1 is log-convex in a_1, the integrand in the above expression is now the product of functions that are log-convex in (a_1, a_2). Hence, the integrand is log-convex and therefore convex in (a_1, a_2). It now follows that expected utility from rewards are concave in the agent's action. These are the main steps in Ábrahám et al.'s (2011) justification of the FOA.

Note that log-convexity of $U'(\cdot)$ plays two roles above. Moreover, log-convexity of $U'(\cdot)$ is equivalent to $U'(\cdot)$ being log-supermodular in (w, a_2). In the current paper, $V_1(w, a_2)$ plays the role of $U'(\cdot)$ in (25). Assumption A5 implies that $V_1(w, a_2)$ is log-supermodular in (w, a_2) (Lemma 2). This assumption is used to discipline the FOA contract in (7). However, since $V_1(w, a_2)$ is not necessarily log-convex in a_2, the above argument cannot be used to establish concavity. Instead, concavity in Lemma 1 comes from the convexity assumption in Assumption A3 (LOCC) and the substitutability assumption that $v_{12} < 0$. In A3, convexity also reduces to requiring that the product of two functions, $G^1(x_1|a_1)$ and $G^2(x_2|a_2)$, are convex in (a_1, a_2). Log-convexity of each function is again sufficient.

A.3 Assumption A4

It is possible to relax the assumption that tasks are substitutes in the cost function, or $c_{12} \geq 0$. The arguments that led to the reduced problem do not depend on this assumption. Hence, even if it is assumed that tasks are complements, or $c_{12} \leq 0$, there is a set of actions on which the reduced problem identifies the optimal contract. Moreover, for a given a_1, there is once again a cut-off value of a_2, $s(a_1|\overline{u})$, such that (P) is redundant if and only if $a_2 \leq s(a_1|\overline{u})$. Theorem 1 applies.

However, the sign of EU_{12} is ambiguous when $c_{12} < 0$. The reason is that tasks are substitutes in expected rewards, yet complements in the cost function. If the latter effect dominates, then implementation costs are strictly decreasing in a_2 for all $a_2 \leq s(a_1|\overline{u})$. This can be seen by using the argument in the proof of Proposition 4. The agent will then be induced to distort his work-life balance further towards life compared to the symmetric-information benchmark and (P) must bind. That is, the agent's private life is too rich compared to the benchmark. Since marginal costs of effort on the job is decreasing in a_2 in this case, it is also possible that incentives are flatter than with the symmetric-information level of

a_2. Theorem 2, however, does require that $EU_{12} < 0$, and therefore it may no longer be valid once c_{12} is allowed to be negative.

The assumption in A4 that $v_{12} < 0$ is important in several places, including very early on in establishing concavity of the agent's expected payoff (Lemma 1). Relaxing this assumption to allow rewards from different sources to be complements is an important topic for future research but it is likely to be technically challenging.

As just mentioned, the reduced problem does not rely on the assumption that $c_{12} \geq 0$. Neither does the next example. In fact, it does not even require Assumption A1 (independence). This example illustrates why Assumption A4 rules out $v_{12} = 0$. Specifically, the *additive model* has an additively separable rewards function which eliminates any direct interaction between rewards from different sources. As a result, the model is not substantially different from the standard model. The point is that the paper's new results stem from interdependencies in the rewards function. The additive model also effectively reproduces the results of the LEN model.

Example 2 (The Additive Model): Assume that

$$v(w, x_2) = u(w) + q(x_2),$$

where u and q are strictly increasing and strictly concave functions. Note that $v_{12} = 0$. Assume that $c(a_1, a_2)$ is strictly increasing and convex. Recall that a_2 determines the distribution of x_2. Hence, let $Q(a_2)$ denote the expectation of $q(x_2)$, given a_2. By Assumptions A2 and A3, $Q(a_2)$ is strictly increasing and strictly concave. Similarly, a_1 determines the distribution of x_1 and thus the distribution of wages. Let ω denote the contract and write $U(a_1|\omega)$ as the expectation of $u(w(x_1))$, given a_1. Thus,

$$EU(a_1, a_2) = U(a_1|\omega) + Q(a_2) - c(a_1, a_2). \tag{26}$$

Note that for a fixed a_1, the agent's optimal a_2 is unique and independent of the contract. In other words, once the principal has decided which a_1 he wishes to induce, a_2 is predetermined and impossible to manipulate. Henceforth, let $a_2(a_1)$

denote the optimal value of a_2, given a_1. The model is now essentially a standard model since the agent's action is effectively one-dimensional. For concreteness,

$$EU(a_1) = U(a_1|\omega) + Q(a_2(a_1)) - c(a_1, a_2(a_1)).$$

Unsurprisingly, the model has standard features. The principal designs the contract to manipulate a_1. He has to respect the participation constraint that

$$U(a_1|\omega) \geq \overline{u} - Q(a_2(a_1)) + c(a_1, a_2(a_1)).$$

It is easy to verify that the right hand side is increasing in a_1. Thus, the agent must be promised higher rewards from labor income to accept a contract that induces higher effort. To induce interior effort a_1 on the job, L-IC$_1$ is

$$\frac{\partial U(a_1|\omega)}{\partial a_1} = c_1(a_1, a_2(a_1)).$$

Again, it can be checked that the right hand side is increasing in a_1. Thus, to induce higher effort on the job, expected utility from rewards must respond more dramatically to changes in effort. These conclusions are entirely standard.

The LEN model produces identical results. The reason is that the agent's certainty equivalent in the LEN model is separable, as in (26). See Kirkegaard (2016) for a more detailed discussion of private rewards in the LEN model. The chief difference is that the LEN model stipulates that contracts are linear, $w(x_1) = \beta + \alpha x_1$, and that the agent's action is to pick the means of normally distributed signals. Thus, $U(a_1|\omega) = \beta + \alpha a_1$ and $\frac{\partial U(a_1|\omega)}{\partial a_1} = \alpha$. Hence, the LEN model has an extremely convenient one-parameter measure of the strength of incentives, α. The higher α is, the harder the agent works on the job. A drawback of the model in the current paper is that it does not have an equally convenient measure of incentives. On the other hand, the private rewards version of the LEN model is not as flexible since a_2 is predetermined once a_1 has been decided upon. ▲

A.4 Assumption A5

As mentioned, Assumption A5 has two important uses. First, it helps establish that optimal contracts are regular. Second, it is instrumental in the proof that (P) is redundant for some actions. In addition, it has a compelling interpretation. Nevertheless, it is technically interesting to imagine that A5 does not hold and that the function $V(w, a_2)$ is weakly more risk averse than the function $[-V_2(w, a_2)]$, or

$$CE_{-V_2}(a_1, a_2|w(\cdot)) \geq CE_V(a_1, a_2|w(\cdot)).$$

Recall that L-IC$_2$ and (P) are

$$CE_{-V_2}(a_1, a_2|w(\cdot)) = L(a_1, a_2) \text{ and } CE_V(a_1, a_2|w(\cdot)) \geq W(a_1, a_2|\overline{u}),$$

respectively, and that

$$W(a_1, a_2|\overline{u}) > L(a_1, a_2) \text{ for } a_2 > s(a_1|\overline{u}).$$

Hence, if (P) is satisfied, then

$$CE_{-V_2}(a_1, a_2|w(\cdot)) \geq CE_V(a_1, a_2|w(\cdot)) \geq W(a_1, a_2|\overline{u}) > L(a_1, a_2) \text{ for } a_2 > s(a_1|\overline{u}),$$

which violates L-IC$_2$. Hence, as in the multiplicative model, no $a_2 > s(a_1|\overline{u})$ can be implemented. In other words, any implementable action must skew the work-life balance away from life compared to the symmetric information benchmark. Of course, without A5, (P) need not be slack. Likewise, it is hard to guarantee that the optimal contract is monotonic for all x_1. However, it still holds that any optimal contract must on average be weakly flatter than the contract that implements the symmetric information level of work-life balance; see Section C of this Online Appendix.

B Examples using the multiplicative model

In the multiplicative model,

$$v(w, x_2) = -m(w)n(x_2),$$

where m and n are strictly negative, strictly increasing, and strictly concave functions. In this section, it is further assumed that

$$m(w) = 2\sqrt{w} - k,$$

where $k > 0$ is a constant. The domain is restricted to $w \in \left[0, \left(\frac{k}{2}\right)^2\right)$, where $w \geq 0$ is to ensure that $m(w)$ is defined. The restriction that $w < \left(\frac{k}{2}\right)^2$ ensures that $m(w)$ is strictly negative. Square-root utility has been used in standard models to derive optimal contracts. See e.g. Jewitt et al (2008) or Kirkegaard (2017). Those techniques are generalized here.

Note that contrary to the set-up in the main paper, $v(w, x_2)$ is defined on a compact set of wages. This raises the possibility that the optimal contract stipulates wages that are at the corner, i.e. that $w(x_1) = 0$ for some x_1.[10] To stay consistent with the analysis in the main paper, additional restrictions are therefore imposed in the following which serve to guarantee that optimal wages are interior and that (7) in the main paper is valid.

Thus, the remainder of the section is structured as follows. First, the optimal contract that satisfies L-IC for any interior action is characterized, under the assumption that the resulting contract yields interior wages. Implementation costs are then easily derived. Second, based on the first step it is straightforward to identify restrictions that guarantee that interior wages are in fact optimal. Third, to move towards a fully solved example, functional forms are then specified for $c(a_1, a_2)$, $n(x_2)$, $g^1(x_1|a_1)$, and $g^2(x_2|a_2)$ as well. This culminates in a full description of the example from the main paper.

[10] It will be verified below that $w(x_1) < \left(\frac{k}{2}\right)^2$ for all x_1.

B.1 Optimal contracts with interior wages

Given a contract $w(\cdot)$, the agent's expected utility from action (a_1, a_2) is

$$\begin{aligned} EU(a_1, a_2|w(\cdot)) &= \int\int v(w(x_1), x_2) g^1(x_1|a_1) g^2(x_2|a_2) dx_1 dx_2 - c(a_1, a_2) \\ &= -\int m(w(x_1)) g^1(x_1|a_1) dx_1 \int n(x_2) g^2(x_2|a_2) dx_2 - c(a_1, a_2), \end{aligned}$$

or, for convenience,

$$EU(a_1, a_2|w(\cdot)) = -M(a_1|w(\cdot)) N(a_2) - c(a_1, a_2), \tag{27}$$

where

$$\begin{aligned} M(a_1|w(\cdot)) &= \int m(w(x_1)) g^1(x_1|a_1) dx_1 < 0, \\ N(a_2) &= \int n(x_2) g^2(x_2|a_2) dx_2 < 0. \end{aligned}$$

For future reference, Assumption A2 (MLRP) implies that $N'(a_2) > 0$ since $n(x_2)$ is assumed to be strictly increasing. The convexity assumption in Assumption A3 (LOCC) implies that $N''(a_2) < 0$.

Note that

$$V(w, a_2) = -m(w) N(a_2),$$

such that the multiplicative model has the special feature that

$$\frac{V_{12}(w, a_2)}{V_1(w, a_2)} = \frac{N'(a_2)}{N(a_2)}$$

is independent of w. Given the assumptions on $m(w)$, it is moreover the case that

$$\frac{1}{V_1(w, a_2)} = -\frac{\sqrt{w}}{N(a_2)}.$$

To proceed, the participation constraint is initially ignored. The optimal contract that satisfies L-IC is derived, under the assumption that wages are interior. This yields the first order condition in (7), with $\lambda = 0$. Given the structure of the multiplicative model, the first order condition can then be written as

$$\sqrt{w(x_1)} = -\mu_1 N(a_2) l^1_{a_1}(x_1|a_1) - \mu_2 N'(a_2), \tag{28}$$

which relies on the endogenous multipliers μ_1 and μ_2. The next step is to quantify these. To do so, note that L-IC$_1$ can be written

$$-\int \left[2\sqrt{w(x_1)} - k\right] l^1_{a_1}(x_1|a_1) N(a_2) g^1(x_1|a_1) dx_1 - c_1(a_1, a_2) = 0.$$

and utilizing (28) then yields

$$\int \left[2\left(\mu_1 N(a_2) l^1_{a_1}(x_1|a_1) + \mu_2 N'(a_2)\right) + k\right] l^1_{a_1}(x_1|a_1) N(a_2) g^1(x_1|a_1) dx_1 - c_1(a_1, a_2) = 0.$$

Since the likelihood-ratio has mean zero, the condition reduces to

$$2\mu_1 N(a_2)^2 \int \left(l^1_{a_1}(x_1|a_1)\right)^2 g^1(x_1|a_1) dx_1 - c_1(a_1, a_2) = 0,$$

which can be solved for μ_1. For convenience, let $\mathcal{I}(a_1)$ denote the Fisher Information or the variance of the likelihood-ratio,

$$\mathcal{I}(a_1) = \int \left(l^1_{a_1}(x_1|a_1)\right)^2 g^1(x_1|a_1) dx_1,$$

such that

$$\mu_1 = \frac{c_1(a_1, a_2)}{2N(a_2)^2 \mathcal{I}(a_1)} > 0. \tag{29}$$

Similarly, L-IC$_2$ is

$$-\int \left[2\sqrt{w(x_1)} - k\right] N'(a_2) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) = 0,$$

or, using (28),

$$\int \left[2\left(\mu_1 N(a_2) l^1_{a_1}(x_1|a_1) + \mu_2 N'(a_2)\right) + k\right] N'(a_2) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) = 0.$$

Once again, since the likelihood-ratio has mean zero, this reduces to

$$[2\mu_2 N'(a_2) + k]\, N'(a_2) - c_2(a_1, a_2) = 0,$$

which implies that

$$\mu_2 = \frac{1}{2N'(a_2)} \left[\frac{c_2(a_1, a_2)}{N'(a_2)} - k \right]. \tag{30}$$

Using (29) and (30) in (28) now finally yield a close-form candidate for the optimal contract, with

$$\sqrt{w(x_1)} = -\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(x_1|a_1) - \frac{1}{2}\left[\frac{c_2(a_1, a_2)}{N'(a_2)} - k \right]. \tag{31}$$

Proceeding under the assumption that wages are interior – sufficient conditions for which are derived in the next subsection – it follows that

$$\begin{aligned} w(x_1) &= \left(-\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(x_1|a_1) - \left[\frac{c_2(a_1, a_2)}{2N'(a_2)} - \frac{k}{2} \right] \right)^2 \\ &= \left(\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} \right)^2 \left(l^1_{a_1}(x_1|a_1) \right)^2 + \left(\frac{c_2(a_1, a_2)}{2N'(a_2)} - \frac{k}{2} \right)^2 \\ &\quad + 2\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(x_1|a_1) \left(\frac{c_2(a_1, a_2)}{2N'(a_2)} - \frac{k}{2} \right) \end{aligned}$$

when interior action (a_1, a_2) is implemented. The expected implementation costs are obtained by taking the expectation over x_1, given a_1. As mentioned, the likelihood-ratio has mean zero and variance $\mathcal{I}(a_1)$. Hence, implementation costs are

$$E\left[w|a_1, a_2\right] = \left(\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} \right)^2 \mathcal{I}(a_1) + \left(\frac{c_2(a_1, a_2)}{2N'(a_2)} - \frac{k}{2} \right)^2$$

or

$$E\left[w|a_1, a_2\right] = \frac{1}{4\mathcal{I}(a_1)} \left(\frac{c_1(a_1, a_2)}{N(a_2)} \right)^2 + \frac{1}{4} \left(\frac{c_2(a_1, a_2)}{N'(a_2)} - k \right)^2. \tag{32}$$

For comparative statics, note that

$$\begin{aligned}\frac{\partial E\left[w|a_1,a_2\right]}{\partial a_1} &= \frac{1}{4}\frac{-\mathcal{I}'(a_1)}{\mathcal{I}(a_1)^2}\left(\frac{c_1(a_1,a_2)}{N(a_2)}\right)^2+\frac{1}{2\mathcal{I}(a_1)}\frac{c_1(a_1,a_2)}{N(a_2)^2}c_{11}(a_1,a_2) \\ &\quad +\frac{1}{2}\left(\frac{c_2(a_1,a_2)}{N'(a_2)}-k\right)\frac{c_{12}(a_1,a_2)}{N'(a_2)}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial E\left[w|a_1,a_2\right]}{\partial a_2} &= \frac{1}{2\mathcal{I}(a_1)}\left(\frac{c_1(a_1,a_2)}{N(a_2)}\right)\frac{c_{12}(a_1,a_2)N(a_2)-c_1(a_1,a_2)N'(a_2)}{N(a_2)^2} \\ &\quad +\frac{1}{2}\left(\frac{c_2(a_1,a_2)}{N'(a_2)}-k\right)\frac{c_{22}(a_1,a_2)N'(a_2)-c_2(a_1,a_2)N''(a_2)}{N'(a_2)^2}.\end{aligned}$$

It will soon be established that the term in the parenthesis involving k is strictly negative. It follows that if $c_1(\underline{a}_1,\underline{a}_2)=0$ and $c_{12}(\underline{a}_1,\underline{a}_2)>0$ then $E\left[w|a_1,a_2\right]$ is strictly decreasing in both a_1 and a_2 in a neighborhood around $(\underline{a}_1,\underline{a}_2)$.[11] Hence, it it trivial to construct examples with this property. Indeed, Example 1 in the main paper has the features that $c_1(\underline{a}_1,\underline{a}_2)=0$ and $c_{12}(\underline{a}_1,\underline{a}_2)>0$.

Away from $(\underline{a}_1,\underline{a}_2)$, note that the derivative of $E\left[w|a_1,a_2\right]$ with respect to a_2 is more likely to be positive the larger $c_{12}(a_1,a_2)$ is, other things being equal. Thus, a functional form for $c(a_1,a_2)$ that allows c_{12} to be large relative to c_1, c_2, and c_{22} will eventually be chosen.

B.2 Parameter restrictions

Recall the restrictions on the domain of $m(\cdot)$ that $w\geq 0$ and that $w<\left(\frac{k}{2}\right)^2$, where the latter is equivalent to the restriction that $m(w)<0$. This section begins by deriving conditions that guarantee that (31) satisfies these restrictions. Then, functional forms for the utility of private rewards, the cost function, and the distribution function are specified and it is explained how parameter values that satisfy the required conditions were chosen to develop Example 1 in the main paper.

First, (31) is feasible only if the right hand side is non-negative. By MLRP,

[11] Recall that the formula for $E[w|a_1,a_2]$ is valid for interior actions but not necessarily for boundary actions.

it is minimized at $x_1 = \underline{x}_1$, where it is noted that $l^1_{a_1}(\underline{x}_1|a_1) < 0$. Hence, it is required that

$$-\frac{c_1(a_1,a_2)}{N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(\underline{x}_1|a_1) - \left[\frac{c_2(a_1,a_2)}{N'(a_2)} - k\right] \geq 0 \tag{33}$$

for all (a_1, a_2). Note that (33) necessitates that the bracketed term is strictly negative. Thus, since $N'(a_2) > 0$, it follows from (30) that $\mu_2 < 0$. Recall that $\mu_1 > 0$. By MLRP, the solution to (31) is strictly increasing in x_1. Hence, for all x_1 except possibly $\underline{x}_1$, it holds that $w(x_1)$ is strictly positive. For future reference, (33) can be reformulated as

$$k \geq \frac{c_1(a_1,a_2)}{N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(\underline{x}_1|a_1) + \frac{c_2(a_1,a_2)}{N'(a_2)} \text{ for all } (a_1,a_2). \tag{34}$$

Second, $m(w(x_1)) = 2\sqrt{w(x_1)} - k < 0$ is hardest to satisfy at $x_1 = \overline{x}_1$. Using (31) then yields the condition that

$$-\frac{c_1(a_1,a_2)}{N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(\overline{x}_1|a_1) - \frac{c_2(a_1,a_2)}{N'(a_2)} < 0 \text{ for all } (a_1,a_2), \tag{35}$$

or

$$c_2(a_1,a_2) > -\frac{N'(a_2)}{N(a_2)} \frac{l^1_{a_1}(\overline{x}_1|a_1)}{\mathcal{I}(a_1)} c_1(a_1,a_2). \tag{36}$$

To present fully solved examples, it is of course necessary to specify functional forms for all the primitives. This is done in the following. Throughout, the support of a_i is taken to be the interval $[\underline{a}_i, \overline{a}_i] = [0,1]$.

THE COST FUNCTION: To begin, consider the relatively flexible form

$$\begin{aligned} c(a_1,a_2) &= t + t_1(a_1 - \widehat{a}_1) + t_2(a_2 - \widehat{a}_2) + \frac{1}{2}t_{11}(a_1 - \widehat{a}_1)^2 + \frac{1}{2}t_{22}(a_2 - \widehat{a}_2)^2 \\ &\quad + t_{12}(a_1 - \widehat{a}_1)(a_2 - \widehat{a}_2). \end{aligned} \tag{37}$$

The example in Kirkegaard (2016) makes use of this functional form, but with different parameter values than in the example provided in the current version of the paper.

The functional form is fairly easy to work with in part because $c_{ij}(a_i, a_j) = t_{ij}$

is constant, $i, j = 1, 2$. It can be viewed as a (second order) Taylor approximation of a general cost function. However, there are evidently a lot of parameters to specify. To help manage this, it will be assumed that $t_{11}t_{22} - t_{12}^2 = 0$, which already eliminates one degree of freedom. As alluded to earlier, it will also be assumed that $c_1(0, 0) = 0$. This eliminates another degree of freedom. It will be explained momentarily how (34) and (36) are used to further restrict the parameters. Parameters $\widehat{a}_i$ were chosen as $\widehat{a}_i = \overline{a}_i = 1$, implying that $c_i(\overline{a}_1, \overline{a}_2) = t_i$. This simply made it easier to search for parameter values where $E\left[w|a_1, a_2\right]$ is increasing in a_i in a neighborhood around $(\overline{a}_1, \overline{a}_2)$. The parameter t is unimportant since implementation costs depend only on marginal costs. Hence, it has been chosen to normalize $c(0, 0) = 0$.

In the solved example, the cost function can be simplified to

$$c(a_1, a_2) = 0.7313a_2 + 0.405a_1^2 + 0.005a_2^2 + 0.09a_1a_2. \tag{38}$$

By continuity, small changes in the parameter values do not change the main properties of the example.

THE MARGINAL DISTRIBUTION FUNCTIONS: Consider the marginal distribution functions

$$G^i(x_i|a_i) = \left(1 - e^{-\frac{a_i+8}{72}}\right)x_i^2 + \left(e^{-\frac{a_i+8}{72}}\right)x_i, \quad x_i \in [0, 1], \quad i = 1, 2.$$

It can be verified that $G^i(x_i|a_i)$ is log-convex in a_i. As explained in Section A.2 of this Online Appendix, log-convexity implies that the joint distribution function $F(x_1, x_2|a_1, a_2)$ satisfies Assumption A3 (LOCC). It is also straightforward to verify that $G^i(x_i|a_i)$ satisfies Assumption A2 (MLRP). This distribution function has the special property that $l_{a_i}^i(\underline{x}_i|a_i)$ is independent of a_i.

It is also possible to calculate Fisher Information in closed form,

$$\mathcal{I}(a_1) = \frac{\left(\ln\left(2e^{\frac{a_1+8}{72}} - 1\right)\right)e^{\frac{a_1+8}{72}} - 2e^{\frac{a_1+8}{72}} + 2}{10\,368e^{3\times\frac{a_1+8}{72}} - 31\,104e^{2\times\frac{a_1+8}{72}} + 31\,104e^{\frac{a_1+8}{72}} - 10\,368} > 0.$$

This example has the property that $\mathcal{I}(a_1)$ is strictly decreasing and that $\frac{l_{a_1}^1(\overline{x}_1|a_1)}{\mathcal{I}(a_1)}$

is strictly increasing in a_1. Given these properties and that the cost function takes the form in (37), the right hand side of (34) is increasing in a_1 and a_2. Hence, the condition is hardest to satisfy at $(\overline{a}_1, \overline{a}_2)$. For concreteness, k is then specified as the value that leads (34) to bind at $(\overline{a}_1, \overline{a}_2)$. This value depends on the function $N(a_2)$, which is specified next.

The private rewards function: It is assumed that

$$n(x_2) = 7.5\sqrt{x_2} - 8,$$

and it can then be verified that

$$N(a_2) = -\left(2 + e^{-\frac{a_2+8}{72}}\right), \quad N'(a_2) = \frac{1}{72} e^{-\frac{a_2+8}{72}}.$$

With this functional form, k as fixed above now takes the value $k = 153.696$. Condition (36) is dealt with in a way that is inspired by how condition (34) was dealt with. In particular, the parameters in the cost function was chosen such that (36) is just satisfied at $(\overline{a}_1, \overline{a}_2)$ and it was then verified that this is sufficient to ensure that the condition is satisfied globally. This eliminates yet another degree of freedom from the cost function.

Finishing the example: As explained above, various restrictions have been used to eliminate several degrees of freedom from (37). To proceed, an arbitrary but strictly positive value of t_{22} was chosen. This is essentially just a normalization. This leaves one degree of freedom, t_{12}. After having experimented with different values of t_{12}, one was chosen that makes it easier to visualize the different comparative statics that are possible, in particular that implementation costs can be locally increasing or locally decreasing in a_1 and in a_2. The resulting cost function is (38).

Implementation costs in Figure 1(a) were obtained by substituting $c(a_1, a_2)$, k, $\mathcal{I}(a_1)$, and $N(a_2)$ into (32). To obtain expected utility in Figure 1(b), $N(a_2)$ and $c(a_1, a_2)$ were used in (27) alongside the value of $M(a_1|w(\cdot))$ that satisfies L-IC$_2$.

Flatter incentives in the example: From (31),

$$\frac{\partial\sqrt{w(x_1)}}{\partial x_1} = \frac{-c_1(a_1,a_2)}{2N(a_2)\mathcal{I}(a_1)}\frac{\partial l^1_{a_1}(x_1|a_1)}{\partial x_1},$$

the last factor of which is strictly positive by Assumption A2 (MLRP). The first factor is strictly increasing in a_2 given $c_{12} \geq 0$ and $N'(a_2) > 0$. Hence, if the optimal wage schedules that implement the same a_1 but different a_2 intersect, then the wage schedule associated with the larger a_2 must be steeper at that point. Therefore, the optimal contracts can cross at most once.

C Flatter incentives

This section formalizes the intuition in Section 4.2 that incentives are flatter when the agent is induced to work less hard.

The standard way of thinking of a contract is as a mapping from the signal, x_1, to the wage, w. However, this is not the only way to think about a contract. First, note that the optimal contract depends on the signal x_1 only through the likelihood-ratio, $l^1_{a_1}(x_1|a_1)$. Second, the utility function $V(\cdot, a_2)$ is used to evaluate the incentives for effort on the job for any given contract. Different utility functions gives different incentives, even for the same wage schedule. Combining these two observations leads to the idea of translating the contract into a mapping from the likelihood-ratio to the agent's utility. In other words, for any given realization of the likelihood-ratio, the contract gives the agent a certain amount of utils. In the current model, however, utility also depends on a_2.

Now fix two contracts, $\widehat{w}$ and $\widetilde{w}$, that optimally induce interior actions $(a_1, \widehat{a}_2)$ and $(a_1, \widetilde{a}_2)$, respectively. Assume $\widehat{a}_2 > \widetilde{a}_2$. It turns out that as long as the agent uses any utility function $V(\cdot, a_2)$ with $a_2 \in [\widetilde{a}_2, \widehat{a}_2]$ to evaluate incentives, there is a sense in which incentives for effort on the job are flatter with $\widetilde{w}$ than with $\widehat{w}$.

Proposition 6 *For any $a_2 \in [\widetilde{a}_2, \widehat{a}_2]$, the covariance between $V(\widetilde{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$ is strictly smaller than the covariance between $V(\widehat{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$.*

Proof. The proof follows almost trivially from L-IC$_1$. By definition, $EU_1(a_1, \widehat{a}_2|\widehat{w}(\cdot)) = 0$. Since $EU_{12} < 0$, it then holds that $EU_1(a_1, a_2|\widehat{w}(\cdot)) > 0$ for all $a_2 < \widehat{a}_2$. By similar reasoning, $EU_1(a_1, a_2|\widetilde{w}(\cdot)) < 0$ for all $a_2 > \widetilde{a}_2$. Combining the two yields

$$\begin{aligned}\int V(\widehat{w}(x_1), a_2) l^1_{a_1}(x_1|a_1) g^1(x_1|a_1) dx_1 &\geq c_1(a_1, a_2) \\ &\geq \int V(\widetilde{w}(x_1), a_2) l^1_{a_1}(x_1|a_1) g^1(x_1|a_1) dx_1,\end{aligned}$$

for all $a_2 \in [\widetilde{a}_2, \widehat{a}_2]$, with at least one strict inequality. Since the likelihood-ratio has mean zero, the first term is the covariance between $V(\widehat{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$, while the last term is the covariance between $V(\widetilde{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$. This concludes the proof. ■

Proposition 6 implies that the slope of a regression of $V(\widetilde{w}(x_1), a_2)$ on $l^1_{a_1}(x_1|a_1)$ is strictly smaller than the slope of a regression of $V(\widehat{w}(x_1), a_2)$ on $l^1_{a_1}(x_1|a_1)$. The regression irons out the non-linearities in the agent's utility and thereby presents a way to think about "average" incentives. The slope can be thought of as the average piece-rate, measured in utils, for a marginal increase in the likelihood-ratio. Future research is planned to pursue other implications of this idea.

D A reinterpretation of the reduced problem

Only L-IC$_1$ and L-IC$_2$ enter the reduced problem. Although these are equality constraint, for the sake of argument imagine weakening L-IC$_2$ by turning it into an inequality constraint, such that the constraints can be written

$$\int V(w(x_1), a_2) g^1_{a_1}(x_1|a_1) dx_1 - c_1(a_1, a_2) = 0 \tag{39}$$

$$\int \left[-V_2(w(x_1), a_2)\right] g^1(x_1|a_1) dx_1 - c(a_1, a_2) \geq -c(a_1, a_2) - c_2(a_1, a_2). \tag{40}$$

In comparison, consider the following contracting problem. First, a_2 is a fixed parameter in the agent's utility function (alternatively, the principal can dictate its value). Second, the agent has a "split personality" when it comes to evaluating the contract. He uses the utility function $V(w, a_2)$ to evaluate incentives but the utility function $-V_2(w, a_2)$ to evaluate the merits of participation. Third, the agent's reservation utility depends on the principal's recommendation, with

$\overline{u}(a_1, a_2) = -c(a_1, a_2) - c_2(a_1, a_2)$ describing this reservation utility. Evidently, (39) and (40) define the incentive compatibility constraint and the participation constraint, respectively, in this particular contracting problem.

Note that reservation utility in this model is strictly decreasing in a_1. Thus, the personality that decides on participation is (i) more risk averse and (ii) experiences some kind of self-satisfaction from working hard, thus lowering the threshold for participation when the agent expects to be working hard. Since this side of the agent's personality is more risk averse, it is more put off by any risk included in the contract. On the other hand, it is "intrinsically motivated" to work hard. If this effect is strong enough, implementation costs may be decreasing in a_1, as illustrated in Example 1 and Proposition 5.

Next, note that the standard argument described in Section 2 can be used to prove that the "participation constraint" must bind. Hence, the inequality constraint effectively becomes an equality constraint, as in the reduced problem. Thus, the two models are essentially equivalent for a fixed action.

E Common agency

Given a_2, the principal considers the distribution of private rewards to be fixed. However, the outside rewards are sometimes derived from other principal-agent relationships. This is the case when the agent holds several jobs. In such cases of common agency, principals are strategically interacting with each other.

Bernheim and Whinston (1986) were first to consider such situations. However, they assume that every principal observes the same information. Thus, any principal can observe and verify how well the agent performed for other principals. Bernheim and Whinston (1986) establish that the equilibrium action is implemented at a total cost that coincides with the total cost that would have obtained if the principals could collude (or merge). As Bernheim and Whinston (1986) explain: "We can always view a principal as constructing his incentive scheme in two steps: he first undoes what all the other principals have offered and then makes an 'aggregate' offer [...]. Clearly, if we are at an equilibrium, each principal must, in this second step, select an aggregate offer that implements the equilibrium action at minimum cost." On the other hand, competition between principals typically distorts the equilibrium action away from the second-best.

The model in the current paper instead assumes that outside rewards are private. That is, any given principal cannot observe how well the agent performs for another principal. Holmström and Milgrom (1988) use the term “disjoint observations” to refer to such a setting.

Holmström and Milgrom (1988) use the LEN model to show that the equilibrium action is implemented in a cost-minimizing manner when signals are independent. That is, given independence, Bernheim and Whinston’s (1986) result on joint observations extends to disjoint observations in the LEN model. The underlying reason is that independence together with linear contracts and exponential utility imply so much “separability” that nothing is gained from collusion.

Now, the current model does not have the benefit of the same degree of separability. A complete analysis of the common agency problem in this setting is outside the scope of the paper but is planned for future research. However, a natural conjecture is that the equilibrium action is implemented at higher than minimum costs. Thus, the model has a source of distortion that is absent in Bernheim and Whinston (1986) and Holmström and Milgrom (1988).