Online Appendix to:
Contracting with Private Rewards

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Overview

The first section of this appendix is devoted to discussing the assumptions of the model. The assumptions are interpreted and in some cases justified more carefully. The relationship to some existing literature is spelled out in more detail and possible relaxations are discussed as well.

The second section describes the details of Example 1 from the main article, using the multiplicative model with square-root utility. It includes a closed-form solution of the optimal contract that induces any given interior action as well as an explicit characterization of implementation costs. The third section argues more formally that incentives are flatter when smaller a_2 are induced.

The fourth section presents a reinterpretation of the reduced problem in which the agent is intrinsically motivated to work hard on the job. The fifth and final section examines the model's link to the literature on common agency.

A Assumptions

This section discusses Assumptions A1–A5 in more detail. An example is given in which A4 does not hold. This example clarifies how the model differs from, and is richer than, the Linear-Exponential-Normal model.

A.1 Assumptions A1–A2

Assumption A1 (independence) assumes that the signal x_1 and the private reward x_2 are independent. For example, there is little reason to think that job performance and the mastery of a hobby are correlated. In other settings, such as when the agent is moonlighting in the same industry, the independence assumption is harder to justify. However, the assumption may have some behavioral justification even in such cases. In particular, there is a growing literature on the prevalence and consequences of *correlation neglect*. See e.g. Levy and Razin (2015) and the references therein. In the current context, correlation neglect arises if the principal and the agent know the marginal distributions, but ignore any correlation between the random variables in the joint distribution.

There are at least two technical problems related to relaxing the independence assumption. One is to establish a counterpart to Lemma 1 for "well-behaved" contracts. Moreover, (7) may no longer apply. Thus, it also becomes harder to verify whether the contract is "well-behaved" in the first place. In short, A1 captures the main price of allowing the rewards function to be non-separable.

Assumption A2 (MLRP) ensures both that (i) the contract is regular and that (ii) a first order stochastic dominance property holds, i.e. that $G^i_{a_i}(x_i|a_i) < 0$ for $x_i \in (\underline{x}_i, \overline{x}_i)$. The assumption that $g^1(x_1|a_1)$ is log-supermodular can be replaced with the assumption that the first order stochastic dominance property holds and that there is an exogenous restriction that the contract must be non-decreasing in x_1. Such a restriction arises if the agent can sabotage the signal after it has been realized but before the principal observes it.

The assumption that $g^2(x_2|a_2)$ is log-supermodular plays a role in the aggregation result in Lemma 2. It can be replaced by the first order stochastic dominance property and the more direct assumption that (15) holds. Note that (15) is automatic in the multiplicative model.

A.2 Assumption A3

Assumption A3 (LOCC) is a technically motivated assumption that is instrumental in justifying the solution method. It is a direct extension of Rogerson's (1985) convexity assumption (CDFC). Recall that Rogerson assumes that there is a single signal and a single task. Conlon (2009) presents justifications of the first-order approach (FOA) that permit multiple signals but a single task. Kirkegaard (2017) allows multiple tasks, under the assumption that A1 holds. However, private rewards are ruled out and all signals are contractible. In the previous version of the current article, Kirkegaard (2016), justifications of the FOA with private rewards are given that are in the spirit of Jewitt's (1988) single-task justifications.

A sufficient condition for LOCC is that G^1 and G^2 are both log-convex. The product of log-convex functions is itself log-convex, and therefore necessarily convex. Alternatively, fix some G^1 that is strictly convex in a_1, but not necessarily log-convex. Then, there is always some "sufficiently convex" G^2 function that ensures that Assumption A3 is satisfied. For example, a non-negative function $h(z)$ is said to be ρ-convex if $h(z)^\rho/\rho$ is convex, or $h''(z)h(z)/h'(z)^2 \geq 1-\rho$ for all z. Thus, a ρ-convex function is log-convex if and only if $\rho \leq 0$ (and convex if and only if $\rho \leq 1$). If $G^2(x_2|a_2)$ satisfies Assumption A2 and is ρ-convex in a_2 (for all x_2) for some small enough ρ (i.e. ρ is negative, but numerically large), then Assumption A3 is satisfied. To see this, note first that the convexity assumption in A3 necessitates that $G^1 G^1_{a_1 a_1}(G^2 G^2_{a_2 a_2}/\left(G^2_{a_2}\right)^2) - \left(G^1_{a_1}\right)^2 \geq 0$ for interior (x_1, x_2). By ρ-convexity, the left hand side is greater than $G^1 G^1_{a_1 a_1}(1-\rho) - \left(G^1_{a_1}\right)^2 \geq 0$. Hence, the inequality is satisfied if ρ is small enough. To reiterate, as long as G^1 satisfies a strict version of CDFC there are G^2 functions that will permit the FOA to be justified even when allowing for private rewards.

There are some similarities between the current model of private rewards and the literature on hidden savings. Ábrahám et al. (2011) consider a situation where the agent works for the principal while simultaneously privately investing in a risk-free asset. There is thus no uncertainty concerning the return to the non-contractible action. Hence, performance on the job, x_1, is the only source of uncertainty. Ábrahám et al. (2011) justify the FOA by assuming that the distribution of x_1 is log-convex in effort on the job, a_1, and that the agent has decreasing absolute risk aversion. Assumptions A3 (LOCC) and A5 (log-supermodularity) in the current article can be seen as extensions that allow returns that are both stochastic and potentially non-monetary.

More specifically, let a_2 denote the dollar amount that the agent saves. Savings has a risk-free rate of return of r. Letting $U(\cdot)$ denote the Bernoulli utility function over total wealth, the agent's utility upon earning $w(x_1)$ on the job and ra_2 from savings is $U(w(x_1) + ra_2)$. Given action (a_1, a_2), integration by parts yields expected utility from rewards of

$$\int U(w(x_1)+ra_2)g^1(x_1|a_1)dx_1 = U(w(\overline{x}_1)+ra_2) - \int U'(w(x_1)+ra_2)w'(x_1)G^1(x_1|a_1)dx_1. \tag{25}$$

The first term is concave in (a_1, a_2), given the agent is risk averse. Next, note that decreasing absolute risk aversion in total income is equivalent to log-convexity of $U'(\cdot)$. First, since $U'(\cdot)$ is log-convex in w, the right hand side of the counterpart to (7) is then well-behaved. Second, $U'(\cdot)$ is log-convex in a_2. Then, assuming G^1 is log-convex in a_1, the integrand in the above expression is now the product of functions that are log-convex in (a_1, a_2). Hence, the integrand is log-convex and therefore convex in (a_1, a_2). It now follows that expected utility from rewards are concave in the agent's action. These are the main steps in Ábrahám et al.'s (2011) justification of the FOA.

Note that log-convexity of $U'(\cdot)$ plays two roles above. Moreover, log-convexity of $U'(\cdot)$ is equivalent to $U'(\cdot)$ being log-supermodular in (w, a_2). In the current article, $V_1(w, a_2)$ plays the role of $U'(\cdot)$ in (25). Assumption A5 implies that $V_1(w, a_2)$ is log-supermodular in (w, a_2) (Lemma 2). This assumption is used to discipline the FOA contract in (7). However, since $V_1(w, a_2)$ is not necessarily log-convex in a_2, the above argument cannot be used to establish concavity. Instead, concavity in Lemma 1 comes from the convexity assumption in Assumption A3 (LOCC) and the substitutability assumption that $v_{12} < 0$. In A3, convexity also reduces to requiring that the product of two functions, $G^1(x_1|a_1)$ and $G^2(x_2|a_2)$, are convex in (a_1, a_2). Log-convexity of each function is again sufficient.

A.3 Assumption A4

It is possible to relax the assumption that tasks are substitutes in the cost function, or $c_{12} \geq 0$. The arguments that led to the reduced problem do not depend on this assumption. Hence, even if it is assumed that tasks are complements, or $c_{12} \leq 0$, there is a set of actions on which the reduced problem identifies the optimal contract. Moreover, for a given a_1, there is once again a cut-off value of a_2, $s(a_1|\overline{u})$, such that (P) is redundant if and only if $a_2 \leq s(a_1|\overline{u})$. Theorem 1 applies.

However, the sign of EU_{12} is ambiguous when $c_{12} < 0$. The reason is that tasks are substitutes in expected rewards, yet complements in the cost function. If the latter effect dominates, then implementation costs are strictly decreasing in a_2 for all $a_2 \leq s(a_1|\overline{u})$. This can be seen by using the argument in the proof of Proposition 4. The agent will then be induced to distort his work-life balance further towards life compared to the symmetric-information benchmark and (P) must bind. That is, the agent's private life is too rich compared to the benchmark. Since marginal costs of effort on the job is decreasing in a_2 in this case, it is also possible that incentives are flatter than with the symmetric-information level of a_2. Theorem 2, however, does require that $EU_{12} < 0$, and therefore it may no longer be valid once c_{12} is allowed to be negative.

The assumption in A4 that $v_{12} < 0$ is important in several places, including very early on in establishing concavity of the agent's expected payoff (Lemma 1). Relaxing this assumption to allow rewards from different sources to be complements is an important topic for future research but it is likely to be technically challenging.

As just mentioned, the reduced problem does not rely on the assumption that $c_{12} \geq 0$. Neither does the next example. In fact, it does not even require Assumption A1 (independence). This example illustrates why Assumption A4 rules out $v_{12} = 0$. Specifically, the *additive model* has an additively separable rewards function which eliminates any direct interaction between rewards from different sources. As a result, the model is not substantially different from the standard model. The point is that the article's new results stem from interdependencies in the rewards function. The additive model also effectively reproduces the results of the LEN model.

Example 2 (The Additive Model): Assume that

$$v(w, x_2) = u(w) + q(x_2),$$

where u and q are strictly increasing and strictly concave functions. Note that $v_{12} = 0$. Assume that $c(a_1, a_2)$ is strictly increasing and convex. Recall that a_2 determines the distribution of x_2. Hence, let $Q(a_2)$ denote the expectation of $q(x_2)$, given a_2. By Assumptions A2 and A3, $Q(a_2)$ is strictly increasing and strictly concave. Similarly, a_1 determines the distribution of x_1 and thus the distribution of wages. Let ω denote the contract and write $U(a_1|\omega)$ as the expectation of $u(w(x_1))$, given a_1. Thus,

$$EU(a_1, a_2) = U(a_1|\omega) + Q(a_2) - c(a_1, a_2). \tag{26}$$

Note that for a fixed a_1, the agent's optimal a_2 is unique and independent of the contract. In other words, once the principal has decided which a_1 he wishes to induce, a_2 is predetermined and impossible to manipulate. Henceforth, let $a_2(a_1)$ denote the optimal value of a_2, given a_1. The model is now essentially a standard model since the agent's action is effectively one-dimensional. For concreteness,

$$EU(a_1) = U(a_1|\omega) + Q(a_2(a_1)) - c(a_1, a_2(a_1)).$$

Unsurprisingly, the model has standard features. The principal designs the contract to manipulate a_1. He has to respect the participation constraint that

$$U(a_1|\omega) \geq \overline{u} - Q(a_2(a_1)) + c(a_1, a_2(a_1)).$$

It is easy to verify that the right hand side is increasing in a_1. Thus, the agent must be promised higher rewards from labor income to accept a contract that induces higher effort. To induce interior effort a_1 on the job, L-IC$_1$ is

$$\frac{\partial U(a_1|\omega)}{\partial a_1} = c_1(a_1, a_2(a_1)).$$

Again, it can be checked that the right hand side is increasing in a_1. Thus, to induce higher effort on the job, expected utility from rewards must respond more dramatically to changes in effort. These conclusions are entirely standard.

The LEN model produces identical results. The reason is that the agent's certainty equivalent in the LEN model is separable, as in (26). See Kirkegaard (2016) for a more detailed discussion of private rewards in the LEN model. The chief difference is that the LEN model stipulates that contracts are linear, $w(x_1) = \beta + \alpha x_1$, and that the agent's action is to pick the means of normally distributed signals. Thus, $U(a_1|\omega) = \beta + \alpha a_1$ and $\frac{\partial U(a_1|\omega)}{\partial a_1} = \alpha$. Hence, the LEN model has an extremely convenient one-parameter measure of the strength of incentives, α. The higher α is, the harder the agent works on the job. A drawback of the model in the current article is that it does not have an equally convenient measure of incentives. On the other hand, the private rewards version of the LEN model is not as flexible since a_2 is predetermined once a_1 has been decided upon. ▲

A.4 Assumption A5

As mentioned, Assumption A5 has two important uses. First, it helps establish that optimal contracts are regular. Second, it is instrumental in the proof that (P) is redundant for some actions. In addition, it has a compelling interpretation. Nevertheless, it is technically interesting to imagine that A5 does not hold and that the function $V(w, a_2)$ is weakly more risk averse than the function $[-V_2(w, a_2)]$, or

$$CE_{-V_2}(a_1, a_2|w(\cdot)) \geq CE_V(a_1, a_2|w(\cdot)).$$

Recall that L-IC$_2$ and (P) are

$$CE_{-V_2}(a_1, a_2|w(\cdot)) = L(a_1, a_2) \text{ and } CE_V(a_1, a_2|w(\cdot)) \geq W(a_1, a_2|\overline{u}),$$

respectively, and that

$$W(a_1, a_2|\overline{u}) > L(a_1, a_2) \text{ for } a_2 > s(a_1|\overline{u}).$$

Hence, if (P) is satisfied, then

$$CE_{-V_2}(a_1, a_2|w(\cdot)) \geq CE_V(a_1, a_2|w(\cdot)) \geq W(a_1, a_2|\overline{u}) > L(a_1, a_2) \text{ for } a_2 > s(a_1|\overline{u}),$$

which violates L-IC$_2$. Hence, as in the multiplicative model, no $a_2 > s(a_1|\overline{u})$ can be implemented. In other words, any implementable action must skew the work-life balance away from life compared to the symmetric information benchmark. Of course, without A5, (P) need not be slack. Likewise, it is hard to guarantee that the optimal contract is monotonic for all x_1. However, it still holds that any optimal contract must on average be weakly flatter than the contract that implements the symmetric information level of work-life balance; see Section C of this Online Appendix.

B Examples using the multiplicative model

In the multiplicative model,

$$v(w, x_2) = -m(w)n(x_2),$$

where m and n are strictly negative, strictly increasing, and strictly concave functions. In this section, it is further assumed that

$$m(w) = 2\sqrt{w} - k,$$

where $k > 0$ is a constant. The domain is restricted to $w \in \left[0, \left(\frac{k}{2}\right)^2\right)$, where $w \geq 0$ is to ensure that $m(w)$ is defined. The restriction that $w < \left(\frac{k}{2}\right)^2$ ensures that $m(w)$ is strictly negative. Square-root utility has been used in standard models to derive optimal contracts. See e.g. Jewitt et al (2008) or Kirkegaard (2017). Those techniques are generalized here.

Note that contrary to the set-up in the main article, $v(w, x_2)$ is defined on a compact set of wages. This raises the possibility that the optimal contract stipulates wages that are at the corner, i.e. that $w(x_1) = 0$ for some x_1.[10] To stay consistent with the analysis in the main article, additional restrictions are therefore imposed in the following which serve to guarantee that optimal wages are interior and that (7) in the main article is valid.

Thus, the remainder of the section is structured as follows. First, the optimal contract that satisfies L-IC for any interior action is characterized, under the assumption that the resulting contract yields interior wages. Implementation costs are then easily derived. Second, based on the first step it is straightforward to identify restrictions that guarantee that interior wages are in fact optimal. Third, to move towards a fully solved example, functional forms are then specified for $c(a_1, a_2)$, $n(x_2)$, $g^1(x_1|a_1)$, and $g^2(x_2|a_2)$ as well. This culminates in a full description of the example from the main article.

[10] It will be verified below that $w(x_1) < \left(\frac{k}{2}\right)^2$ for all x_1.

B.1 Optimal contracts with interior wages

Given a contract $w(\cdot)$, the agent's expected utility from action (a_1, a_2) is

$$\begin{aligned} EU(a_1, a_2|w(\cdot)) &= \int\int v(w(x_1), x_2) g^1(x_1|a_1) g^2(x_2|a_2) dx_1 dx_2 - c(a_1, a_2) \\ &= -\int m(w(x_1)) g^1(x_1|a_1) dx_1 \int n(x_2) g^2(x_2|a_2) dx_2 - c(a_1, a_2), \end{aligned}$$

or, for convenience,

$$EU(a_1, a_2|w(\cdot)) = -M(a_1|w(\cdot))N(a_2) - c(a_1, a_2), \tag{27}$$

where

$$\begin{aligned} M(a_1|w(\cdot)) &= \int m(w(x_1)) g^1(x_1|a_1) dx_1 < 0, \\ N(a_2) &= \int n(x_2) g^2(x_2|a_2) dx_2 < 0. \end{aligned}$$

For future reference, Assumption A2 (MLRP) implies that $N'(a_2) > 0$ since $n(x_2)$ is assumed to be strictly increasing. The convexity assumption in Assumption A3 (LOCC) implies that $N''(a_2) < 0$.

Note that

$$V(w, a_2) = -m(w)N(a_2),$$

such that the multiplicative model has the special feature that

$$\frac{V_{12}(w, a_2)}{V_1(w, a_2)} = \frac{N'(a_2)}{N(a_2)}$$

is independent of w. Given the assumptions on $m(w)$, it is moreover the case that

$$\frac{1}{V_1(w, a_2)} = -\frac{\sqrt{w}}{N(a_2)}.$$

To proceed, the participation constraint is initially ignored. The optimal contract that satisfies L-IC is derived, under the assumption that wages are interior. This yields the first order condition in (7), with $\lambda = 0$. Given the structure of the multiplicative model, the first order condition can then be written as

$$\sqrt{w(x_1)} = -\mu_1 N(a_2) l^1_{a_1}(x_1|a_1) - \mu_2 N'(a_2), \tag{28}$$

which relies on the endogenous multipliers μ_1 and μ_2. The next step is to quantify these. To do so, note that L-IC$_1$ can be written

$$-\int \left[2\sqrt{w(x_1)} - k\right] l^1_{a_1}(x_1|a_1) N(a_2) g^1(x_1|a_1) dx_1 - c_1(a_1, a_2) = 0.$$

and utilizing (28) then yields

$$\int \left[2\left(\mu_1 N(a_2) l^1_{a_1}(x_1|a_1) + \mu_2 N'(a_2)\right) + k\right] l^1_{a_1}(x_1|a_1) N(a_2) g^1(x_1|a_1) dx_1 - c_1(a_1, a_2) = 0.$$

Since the likelihood-ratio has mean zero, the condition reduces to

$$2\mu_1 N(a_2)^2 \int \left(l^1_{a_1}(x_1|a_1)\right)^2 g^1(x_1|a_1) dx_1 - c_1(a_1, a_2) = 0,$$

which can be solved for μ_1. For convenience, let $\mathcal{I}(a_1)$ denote the Fisher Information or the variance of the likelihood-ratio,

$$\mathcal{I}(a_1) = \int \left(l^1_{a_1}(x_1|a_1)\right)^2 g^1(x_1|a_1) dx_1,$$

such that

$$\mu_1 = \frac{c_1(a_1, a_2)}{2N(a_2)^2 \mathcal{I}(a_1)} > 0. \tag{29}$$

Similarly, L-IC$_2$ is

$$-\int \left[2\sqrt{w(x_1)} - k\right] N'(a_2) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) = 0,$$

or, using (28),

$$\int \left[2\left(\mu_1 N(a_2) l^1_{a_1}(x_1|a_1) + \mu_2 N'(a_2)\right) + k\right] N'(a_2) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) = 0.$$

Once again, since the likelihood-ratio has mean zero, this reduces to

$$[2\mu_2 N'(a_2) + k]\, N'(a_2) - c_2(a_1, a_2) = 0,$$

which implies that

$$\mu_2 = \frac{1}{2N'(a_2)} \left[\frac{c_2(a_1, a_2)}{N'(a_2)} - k \right]. \tag{30}$$

Using (29) and (30) in (28) now finally yield a close-form candidate for the optimal contract, with

$$\sqrt{w(x_1)} = -\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(x_1|a_1) - \frac{1}{2} \left[\frac{c_2(a_1, a_2)}{N'(a_2)} - k \right]. \tag{31}$$

Proceeding under the assumption that wages are interior – sufficient conditions for which are derived in the next subsection – it follows that

$$\begin{aligned} w(x_1) &= \left(-\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(x_1|a_1) - \left[\frac{c_2(a_1, a_2)}{2N'(a_2)} - \frac{k}{2} \right] \right)^2 \\ &= \left(\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} \right)^2 \left(l^1_{a_1}(x_1|a_1) \right)^2 + \left(\frac{c_2(a_1, a_2)}{2N'(a_2)} - \frac{k}{2} \right)^2 \\ &\quad + 2\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(x_1|a_1) \left(\frac{c_2(a_1, a_2)}{2N'(a_2)} - \frac{k}{2} \right) \end{aligned}$$

when interior action (a_1, a_2) is implemented. The expected implementation costs are obtained by taking the expectation over x_1, given a_1. As mentioned, the likelihood-ratio has mean zero and variance $\mathcal{I}(a_1)$. Hence, implementation costs are

$$E[w|a_1, a_2] = \left(\frac{c_1(a_1, a_2)}{2N(a_2)\mathcal{I}(a_1)} \right)^2 \mathcal{I}(a_1) + \left(\frac{c_2(a_1, a_2)}{2N'(a_2)} - \frac{k}{2} \right)^2$$

or

$$E[w|a_1, a_2] = \frac{1}{4\mathcal{I}(a_1)} \left(\frac{c_1(a_1, a_2)}{N(a_2)} \right)^2 + \frac{1}{4} \left(\frac{c_2(a_1, a_2)}{N'(a_2)} - k \right)^2. \tag{32}$$

For comparative statics, note that

$$\begin{aligned}\frac{\partial E\left[w|a_1,a_2\right]}{\partial a_1} &= \frac{1}{4}\frac{-\mathcal{I}'(a_1)}{\mathcal{I}(a_1)^2}\left(\frac{c_1(a_1,a_2)}{N(a_2)}\right)^2+\frac{1}{2\mathcal{I}(a_1)}\frac{c_1(a_1,a_2)}{N(a_2)^2}c_{11}(a_1,a_2)\\ &\quad+\frac{1}{2}\left(\frac{c_2(a_1,a_2)}{N'(a_2)}-k\right)\frac{c_{12}(a_1,a_2)}{N'(a_2)}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial E\left[w|a_1,a_2\right]}{\partial a_2} &= \frac{1}{2\mathcal{I}(a_1)}\left(\frac{c_1(a_1,a_2)}{N(a_2)}\right)\frac{c_{12}(a_1,a_2)N(a_2)-c_1(a_1,a_2)N'(a_2)}{N(a_2)^2}\\ &\quad+\frac{1}{2}\left(\frac{c_2(a_1,a_2)}{N'(a_2)}-k\right)\frac{c_{22}(a_1,a_2)N'(a_2)-c_2(a_1,a_2)N''(a_2)}{N'(a_2)^2}.\end{aligned}$$

It will soon be established that the term in the parenthesis involving k is strictly negative. It follows that if $c_1(\underline{a}_1,\underline{a}_2)=0$ and $c_{12}(\underline{a}_1,\underline{a}_2)>0$ then $E\left[w|a_1,a_2\right]$ is strictly decreasing in both a_1 and a_2 in a neighborhood around $(\underline{a}_1,\underline{a}_2)$.[11] Hence, it it trivial to construct examples with this property. Indeed, Example 1 in the main article has the features that $c_1(\underline{a}_1,\underline{a}_2)=0$ and $c_{12}(\underline{a}_1,\underline{a}_2)>0$.

Away from $(\underline{a}_1,\underline{a}_2)$, note that the derivative of $E\left[w|a_1,a_2\right]$ with respect to a_2 is more likely to be positive the larger $c_{12}(a_1,a_2)$ is, other things being equal. Thus, a functional form for $c(a_1,a_2)$ that allows c_{12} to be large relative to c_1, c_2, and c_{22} will eventually be chosen.

B.2 Parameter restrictions

Recall the restrictions on the domain of $m(\cdot)$ that $w\geq 0$ and that $w<\left(\frac{k}{2}\right)^2$, where the latter is equivalent to the restriction that $m(w)<0$. This section begins by deriving conditions that guarantee that (31) satisfies these restrictions. Then, functional forms for the utility of private rewards, the cost function, and the distribution function are specified and it is explained how parameter values that satisfy the required conditions were chosen to develop Example 1 in the main article.

First, (31) is feasible only if the right hand side is non-negative. By MLRP,

[11] Recall that the formula for $E[w|a_1,a_2]$ is valid for interior actions but not necessarily for boundary actions.

it is minimized at $x_1 = \underline{x}_1$, where it is noted that $l^1_{a_1}(\underline{x}_1|a_1) < 0$. Hence, it is required that

$$-\frac{c_1(a_1,a_2)}{N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(\underline{x}_1|a_1) - \left[\frac{c_2(a_1,a_2)}{N'(a_2)} - k\right] \geq 0 \tag{33}$$

for all (a_1, a_2). Note that (33) necessitates that the bracketed term is strictly negative. Thus, since $N'(a_2) > 0$, it follows from (30) that $\mu_2 < 0$. Recall that $\mu_1 > 0$. By MLRP, the solution to (31) is strictly increasing in x_1. Hence, for all x_1 except possibly $\underline{x}_1$, it holds that $w(x_1)$ is strictly positive. For future reference, (33) can be reformulated as

$$k \geq \frac{c_1(a_1,a_2)}{N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(\underline{x}_1|a_1) + \frac{c_2(a_1,a_2)}{N'(a_2)} \text{ for all } (a_1,a_2). \tag{34}$$

Second, $m(w(x_1)) = 2\sqrt{w(x_1)} - k < 0$ is hardest to satisfy at $x_1 = \overline{x}_1$. Using (31) then yields the condition that

$$-\frac{c_1(a_1,a_2)}{N(a_2)\mathcal{I}(a_1)} l^1_{a_1}(\overline{x}_1|a_1) - \frac{c_2(a_1,a_2)}{N'(a_2)} < 0 \text{ for all } (a_1,a_2), \tag{35}$$

or

$$c_2(a_1,a_2) > -\frac{N'(a_2)}{N(a_2)} \frac{l^1_{a_1}(\overline{x}_1|a_1)}{\mathcal{I}(a_1)} c_1(a_1,a_2). \tag{36}$$

To present fully solved examples, it is of course necessary to specify functional forms for all the primitives. This is done in the following. Throughout, the support of a_i is taken to be the interval $[\underline{a}_i, \overline{a}_i] = [0,1]$.

THE COST FUNCTION: To begin, consider the relatively flexible form

$$\begin{aligned} c(a_1,a_2) &= t + t_1(a_1 - \widehat{a}_1) + t_2(a_2 - \widehat{a}_2) + \frac{1}{2}t_{11}(a_1 - \widehat{a}_1)^2 + \frac{1}{2}t_{22}(a_2 - \widehat{a}_2)^2 \\ &\quad + t_{12}(a_1 - \widehat{a}_1)(a_2 - \widehat{a}_2). \end{aligned} \tag{37}$$

The example in Kirkegaard (2016) makes use of this functional form, but with different parameter values than in the example provided in the current version of the article.

The functional form is fairly easy to work with in part because $c_{ij}(a_i, a_j) = t_{ij}$

is constant, $i, j = 1, 2$. It can be viewed as a (second order) Taylor approximation of a general cost function. However, there are evidently a lot of parameters to specify. To help manage this, it will be assumed that $t_{11}t_{22} - t_{12}^2 = 0$, which already eliminates one degree of freedom. As alluded to earlier, it will also be assumed that $c_1(0,0) = 0$. This eliminates another degree of freedom. It will be explained momentarily how (34) and (36) are used to further restrict the parameters. Parameters $\widehat{a}_i$ were chosen as $\widehat{a}_i = \overline{a}_i = 1$, implying that $c_i(\overline{a}_1, \overline{a}_2) = t_i$. This simply made it easier to search for parameter values where $E[w|a_1, a_2]$ is increasing in a_i in a neighborhood around $(\overline{a}_1, \overline{a}_2)$. The parameter t is unimportant since implementation costs depend only on marginal costs. Hence, it has been chosen to normalize $c(0,0) = 0$.

In the solved example, the cost function can be simplified to

$$c(a_1, a_2) = 0.7313a_2 + 0.405a_1^2 + 0.005a_2^2 + 0.09a_1a_2. \tag{38}$$

By continuity, small changes in the parameter values do not change the main properties of the example.

The marginal distribution functions: Consider the marginal distribution functions

$$G^i(x_i|a_i) = \left(1 - e^{-\frac{a_i+8}{72}}\right)x_i^2 + \left(e^{-\frac{a_i+8}{72}}\right)x_i, \quad x_i \in [0,1], \quad i = 1, 2.$$

It can be verified that $G^i(x_i|a_i)$ is log-convex in a_i. As explained in Section A.2 of this Online Appendix, log-convexity implies that the joint distribution function $F(x_1, x_2|a_1, a_2)$ satisfies Assumption A3 (LOCC). It is also straightforward to verify that $G^i(x_i|a_i)$ satisfies Assumption A2 (MLRP). This distribution function has the special property that $l^i_{a_i}(\underline{x}_i|a_i)$ is independent of a_i.

It is also possible to calculate Fisher Information in closed form,

$$\mathcal{I}(a_1) = \frac{\left(\ln\left(2e^{\frac{a_1+8}{72}} - 1\right)\right)e^{\frac{a_1+8}{72}} - 2e^{\frac{a_1+8}{72}} + 2}{10\,368e^{3\times\frac{a_1+8}{72}} - 31\,104e^{2\times\frac{a_1+8}{72}} + 31\,104e^{\frac{a_1+8}{72}} - 10\,368} > 0.$$

This example has the property that $\mathcal{I}(a_1)$ is strictly decreasing and that $\frac{l^1_{a_1}(\overline{x}_1|a_1)}{\mathcal{I}(a_1)}$

is strictly increasing in a_1. Given these properties and that the cost function takes the form in (37), the right hand side of (34) is increasing in a_1 and a_2. Hence, the condition is hardest to satisfy at $(\overline{a}_1, \overline{a}_2)$. For concreteness, k is then specified as the value that leads (34) to bind at $(\overline{a}_1, \overline{a}_2)$. This value depends on the function $N(a_2)$, which is specified next.

THE PRIVATE REWARDS FUNCTION: It is assumed that

$$n(x_2) = 7.5\sqrt{x_2} - 8,$$

and it can then be verified that

$$N(a_2) = -\left(2 + e^{-\frac{a_2+8}{72}}\right), \quad N'(a_2) = \frac{1}{72}e^{-\frac{a_2+8}{72}}.$$

With this functional form, k as fixed above now takes the value $k = 153.696$. Condition (36) is dealt with in a way that is inspired by how condition (34) was dealt with. In particular, the parameters in the cost function was chosen such that (36) is just satisfied at $(\overline{a}_1, \overline{a}_2)$ and it was then verified that this is sufficient to ensure that the condition is satisfied globally. This eliminates yet another degree of freedom from the cost function.

FINISHING THE EXAMPLE: As explained above, various restrictions have been used to eliminate several degrees of freedom from (37). To proceed, an arbitrary but strictly positive value of t_{22} was chosen. This is essentially just a normalization. This leaves one degree of freedom, t_{12}. After having experimented with different values of t_{12}, one was chosen that makes it easier to visualize the different comparative statics that are possible, in particular that implementation costs can be locally increasing or locally decreasing in a_1 and in a_2. The resulting cost function is (38).

Implementation costs in Figure 1(a) were obtained by substituting $c(a_1, a_2)$, k, $\mathcal{I}(a_1)$, and $N(a_2)$ into (32). To obtain expected utility in Figure 1(b), $N(a_2)$ and $c(a_1, a_2)$ were used in (27) alongside the value of $M(a_1|w(\cdot))$ that satisfies L-IC$_2$.

Flatter incentives in the example: From (31),

$$\frac{\partial\sqrt{w(x_1)}}{\partial x_1} = \frac{-c_1(a_1,a_2)}{2N(a_2)\mathcal{I}(a_1)}\frac{\partial l^1_{a_1}(x_1|a_1)}{\partial x_1},$$

the last factor of which is strictly positive by Assumption A2 (MLRP). The first factor is strictly increasing in a_2 given $c_{12} \geq 0$ and $N'(a_2) > 0$. Hence, if the optimal wage schedules that implement the same a_1 but different a_2 intersect, then the wage schedule associated with the larger a_2 must be steeper at that point. Therefore, the optimal contracts can cross at most once.

C Flatter incentives

This section formalizes the intuition that incentives are flatter when the agent is induced to work less hard.

The standard way of thinking of a contract is as a mapping from the signal, x_1, to the wage, w. However, this is not the only way to think about a contract. First, note that the optimal contract depends on the signal x_1 only through the likelihood-ratio, $l^1_{a_1}(x_1|a_1)$. Second, the utility function $V(\cdot, a_2)$ is used to evaluate the incentives for effort on the job for any given contract. Different utility functions gives different incentives, even for the same wage schedule. Combining these two observations leads to the idea of translating the contract into a mapping from the likelihood-ratio to the agent's utility. In other words, for any given realization of the likelihood-ratio, the contract gives the agent a certain amount of utils. In the current model, however, utility also depends on a_2.

Now fix two contracts, $\widehat{w}$ and $\widetilde{w}$, that optimally induce interior actions $(a_1, \widehat{a}_2)$ and $(a_1, \widetilde{a}_2)$, respectively. Assume $\widehat{a}_2 > \widetilde{a}_2$. It turns out that as long as the agent uses any utility function $V(\cdot, a_2)$ with $a_2 \in [\widetilde{a}_2, \widehat{a}_2]$ to evaluate incentives, there is a sense in which incentives for effort on the job are flatter with $\widetilde{w}$ than with $\widehat{w}$.

Proposition 6 *For any $a_2 \in [\widetilde{a}_2, \widehat{a}_2]$, the covariance between $V(\widetilde{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$ is strictly smaller than the covariance between $V(\widehat{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$.*

Proof. The proof follows almost trivially from L-IC$_1$. By definition, $EU_1(a_1, \widehat{a}_2|\widehat{w}(\cdot)) = 0$. Since $EU_{12} < 0$, it then holds that $EU_1(a_1, a_2|\widehat{w}(\cdot)) > 0$ for all $a_2 < \widehat{a}_2$. By similar reasoning, $EU_1(a_1, a_2|\widetilde{w}(\cdot)) < 0$ for all $a_2 > \widetilde{a}_2$. Combining the two yields

$$\begin{aligned}\int V(\widehat{w}(x_1), a_2) l^1_{a_1}(x_1|a_1) g^1(x_1|a_1) dx_1 &\geq c_1(a_1, a_2) \\ &\geq \int V(\widetilde{w}(x_1), a_2) l^1_{a_1}(x_1|a_1) g^1(x_1|a_1) dx_1,\end{aligned}$$

for all $a_2 \in [\widetilde{a}_2, \widehat{a}_2]$, with at least one strict inequality. Since the likelihood-ratio has mean zero, the first term is the covariance between $V(\widehat{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$, while the last term is the covariance between $V(\widetilde{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$. This concludes the proof. ■

Proposition 6 implies that the slope of a regression of $V(\widetilde{w}(x_1), a_2)$ on $l^1_{a_1}(x_1|a_1)$ is strictly smaller than the slope of a regression of $V(\widehat{w}(x_1), a_2)$ on $l^1_{a_1}(x_1|a_1)$. The regression irons out the non-linearities in the agent's utility and thereby presents a way to think about "average" incentives. The slope can be thought of as the average piece-rate, measured in utils, for a marginal increase in the likelihood-ratio. Future research is planned to pursue other implications of this idea.

D A reinterpretation of the reduced problem

Only L-IC$_1$ and L-IC$_2$ enter the reduced problem. Although these are equality constraint, for the sake of argument imagine weakening L-IC$_2$ by turning it into an inequality constraint, such that the constraints can be written

$$\int V(w(x_1), a_2) g^1_{a_1}(x_1|a_1) dx_1 - c_1(a_1, a_2) = 0 \tag{39}$$

$$\int [-V_2(w(x_1), a_2)] g^1(x_1|a_1) dx_1 - c(a_1, a_2) \geq -c(a_1, a_2) - c_2(a_1, a_2). \tag{40}$$

In comparison, consider the following contracting problem. First, a_2 is a fixed parameter in the agent's utility function (alternatively, the principal can dictate its value). Second, the agent has a "split personality" when it comes to evaluating the contract. He uses the utility function $V(w, a_2)$ to evaluate incentives but the utility function $-V_2(w, a_2)$ to evaluate the merits of participation. Third, the agent's reservation utility depends on the principal's recommendation, with

$\overline{u}(a_1, a_2) = -c(a_1, a_2) - c_2(a_1, a_2)$ describing this reservation utility. Evidently, (39) and (40) define the incentive compatibility constraint and the participation constraint, respectively, in this particular contracting problem.

Note that reservation utility in this model is strictly decreasing in a_1. Thus, the personality that decides on participation is (i) more risk averse and (ii) experiences some kind of self-satisfaction from working hard, thus lowering the threshold for participation when the agent expects to be working hard. Since this side of the agent's personality is more risk averse, it is more put off by any risk included in the contract. On the other hand, it is "intrinsically motivated" to work hard. If this effect is strong enough, implementation costs may be decreasing in a_1, as illustrated in Example 1 and Proposition 5.

Next, note that the standard argument described in Section 2 can be used to prove that the "participation constraint" must bind. Hence, the inequality constraint effectively becomes an equality constraint, as in the reduced problem. Thus, the two models are essentially equivalent for a fixed action.

E Common agency

Given a_2, the principal considers the distribution of private rewards to be fixed. However, the outside rewards are sometimes derived from other principal-agent relationships. This is the case when the agent holds several jobs. In such cases of common agency, principals are strategically interacting with each other.

Bernheim and Whinston (1986) were first to consider such situations. However, they assume that every principal observes the same information. Thus, any principal can observe and verify how well the agent performed for other principals. Bernheim and Whinston (1986) establish that the equilibrium action is implemented at a total cost that coincides with the total cost that would have obtained if the principals could collude (or merge). As Bernheim and Whinston (1986) explain: "We can always view a principal as constructing his incentive scheme in two steps: he first undoes what all the other principals have offered and then makes an 'aggregate' offer [...]. Clearly, if we are at an equilibrium, each principal must, in this second step, select an aggregate offer that implements the equilibrium action at minimum cost." On the other hand, competition between principals typically distorts the equilibrium action away from the second-best.

The model in the current article instead assumes that outside rewards are private. That is, any given principal cannot observe how well the agent performs for another principal. Holmström and Milgrom (1988) use the term "disjoint observations" to refer to such a setting.

Holmström and Milgrom (1988) use the LEN model to show that the equilibrium action is implemented in a cost-minimizing manner when signals are independent. That is, given independence, Bernheim and Whinston's (1986) result on joint observations extends to disjoint observations in the LEN model. The underlying reason is that independence together with linear contracts and exponential utility imply so much "separability" that nothing is gained from collusion.

Now, the current model does not have the benefit of the same degree of separability. A complete analysis of the common agency problem in this setting is outside the scope of the article but is planned for future research. However, a natural conjecture is that the equilibrium action is implemented at higher than minimum costs. Thus, the model has a source of distortion that is absent in Bernheim and Whinston (1986) and Holmström and Milgrom (1988).

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