Contracting with Private Rewards*

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Abstract

The canonical moral hazard model is extended to allow the agent to face endogenous and non-contractible uncertainty. The agent works for the principal and simultaneously pursues outside rewards. The contract offered by the principal thus manipulates the agent's "work-life balance." The participation constraint is redundant whenever the work-life ratio is skewed away from "life" compared to a symmetric information benchmark. Then, the agent's expected utility is high and he faces flatter incentives. Such contracts may be optimal when the two activities are strong substitutes in the agent's cost function. Finally, implementation costs may be non-monotonic in effort on the job.

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1 Introduction

The principal-agent model has been tremendously influential in economics. However, by treating leisure as an exogenous black box, the canonical model in many ways ignores that a rich part of the agent's life takes place outside the office. In reality, workers are not just passive consumers when they are off the job. For instance, Bryson and MacKerron (2017) document that the leisure activities that people are the most happy engaging in all involve physical or mental exertion.[1] More broadly, non-work activities include home production, see e.g. Becker (1965). The agent devotes time and attention to a range of activities from the mundane to the extraordinary, like a complicated do-it-yourself renovation. Moonlighting represents yet another relevant activity. Note that the rise of the sharing economy adds ever more opportunities to earn income outside the job.

Hence, it is more accurate to think of the agent as multi-tasking: He invests effort in two arenas, summarized here as "work" and "life". In this paper, the latter more broadly refers to whatever activities the agent is pursuing when he is not working for the principal (leisure, home production, moonlighting).

This paper proposes a multi-tasking model in which the agent decides how hard to work on the job and how hard to pursue non-contractible outside rewards. Rewards from the two sources are substitutes. Likewise, effort devoted to the two activities are substitutes in the agent's cost function. A key point then is that the contract manipulates the agent's entire work-life balance. This may have important consequences. For instance, the standard model predicts a punitive contract that gives the agent only his reservation utility. Such a contract may spur him to aggressively pursue outside rewards and to lower his work effort. That is, "life" substitutes for "work." It may be advantageous to grant the agent higher utility from work. His attention then shifts back towards work and away from life. Flatter incentives are now needed to maintain his focus at work.

Thus, it will be argued that endogenizing the agent's work-life balance may help explain puzzles presented by the standard model and may indeed help shed light on issues that existing models are not really geared to examine.

[1] Examples include going to a concert, exercising, gardening, hiking, pursuing a hobby, etc. Passive activities such as watching TV or browsing the internet are much further down the list.

First, a common criticism is that real world incentives appear weaker than what the standard model suggests. Englmaier and Leider (2012) develop a reciprocity-based model in which the agent, when offered a generous contract, becomes “intrinsically motivated” to work hard in order to repay the principal’s kindness. Thus, flatter extrinsic incentives are needed. Although the current paper is not about extrinsic and intrinsic motivation in this particular sense, optimal contracts share some similarities in the two models.[2] In particular, a competing explanation for why incentives may be weaker in the real world is offered.

Second, “work-life balance” is a worthy topic of study in its own right. At the time of writing, a Google search on the phrase returns over 45 million results. Nevertheless, the standard one-task model ignores the issue. Similarly, the dominant multi-tasking model is structured too rigidly to address the topic in a completely satisfactory way. More concretely, the Linear-Exponential-Normal (LEN) model due to Holmström and Milgrom (1987, 1991) can allow for private rewards but only in such a way that there is essentially no interaction between labor rewards and private rewards.[3] As a result, for any *given* level of effort on the job, the principal is *unable* to manipulate how hard the agent pursues private rewards. Thus, in this application the LEN model is hardly a multi-dimensional model after all. In contrast, in the current model the principal has more flexibility to distort the agent’s effort on the private activity separately from effort on the job. Consequently, the paper provides a more nuanced understanding of when it is optimal to skew the work-life balance towards, or away from, “life.”

Holmström and Milgrom (1991) discuss an example in which a teacher teaches both “basic skills” and “higher-order thinking skills.” The former can be tested, thus yielding a contractible signal. The latter cannot be measured or directly rewarded, yet the teacher may experience some degree of satisfaction from teaching those higher-order skills. Note that the principal (the school board or the general public) cares directly about effort on both tasks in this example. The model presented here allows for that possibility as well. Now, Holmström and Milgrom (1991) argue that the optimal contract rewards good test scores less than might

[2] See Kőszegi (2014) for a survey on “behavioral contract theory” and Bénabou and Tirole (2003) for a different take on intrinsic and extrinsic motivation.

[3] More formally, the agent’s certainty equivalent is additively separable in expected rewards from the two sources.

be expected. The reason is that in their model it is possible to incentivize more effort on teaching higher-order skills *only* if the teacher is simultaneously incentivized to lower effort on teaching basic skills. Thus, it is necessary to sacrifice basic skills for higher-order skills. As was alluded to earlier, this restriction need not apply in the current model. Indeed, it may be possible to incentivize the teacher to exert more effort along both dimensions at the same time.

Thus, the two models yield flat incentives for different reasons and under different circumstances. In Holmström and Milgrom (1991), flat incentives discourage effort on the task that yields a contractible signal and the agent then substitutes effort towards the other task. In the current model, flatter incentives can also be used when the principal wishes to maintain a constant effort on the former task but to independently induce lower effort on the latter task.

To understand the intuition more clearly, consider again the work-life balance interpretation of the model and assume that the principal does not directly care about how hard the agent pursues private rewards. Why then should the principal concern himself with the agent's work-life balance? First, the agent's rewards from outside activities impact how hard it is to incentivize him on the job. Simply put, an agent who does not have much else going on in his life only needs relatively low-powered incentives in order to work hard on the job. Insofar as it is cheaper to provide weaker incentives, this works to the principal's advantage. On the other hand, to prevent the agent from strenuously pursuing outside rewards, the job itself has to be a good source of utility. However, this tends to be costly for the principal. More succinctly, it is costly to incentivize the agent to divest from "life" but doing so makes it cheaper to incentivize him to work hard on the job.

Thus, there is a trade-off. However, it is possible that the agent may perceive the job to be a better source of utility even with a lower expected wage. This may occur if the amount of risk decreases enough, as might be the case when weaker incentives are used. In fact, it is profitable to manipulate and skew the agent's work-life balance away from life when the agent's marginal cost of effort on the job is very sensitive to his effort on the outside activity. Inducing less interest in outside rewards then markedly lowers the agent's marginal costs of effort on the job and incentives can then be made much flatter.

A third and final point is that standard models predict a binding participation constraint. This is arguably somewhat puzzling in light of amble evidence that employed people are happier than unemployed people, see e.g. Clark and Oswald (1994). In contrast, recall that the agent in the current model relaxes his pursuit of private rewards only if he perceives the job to be a rich source of utility. As a consequence, the participation constraint is redundant, and slack, whenever the agent is incentivized to skew his work-life balance away from life compared to what would be optimal under symmetric information. In this case, the agent then experiences high utility and faces weaker incentives. A similar conclusion obtains in Englmaier and Leider (2012), but for entirely different reasons.

In Englmaier and Leider (2012), the agent may be awarded high utility in order to trigger feelings of reciprocity. A change in government policy that increases the value of the outside option makes the principal seem relatively less generous. The agent then becomes less intrinsically motivated and the principal will typically want to change the contract. In standard models, contract design is also very sensitive to the outside option since the participation constraint is binding. In the current model, a small change in reservation utility does not impact the agent's action when the participation constraint is redundant. Hence, there is no need to change the contract. Thus, the terms of the contract, and therefore the principals' wage bill, is less sensitive to changes in the outside option.

The discussion has focused mostly on how the principal manipulates the agent's pursuit of outside rewards. Of course, it is equally important to determine how hard to induce him to work on the job. The standard model yield the intuitive conclusion that implementation costs are increasing in effort on the job. A more technical result is that this need not hold in the model proposed here. There are two competing effects when stimulating higher effort at work. First, steeper incentives are required. On the other hand, it becomes easier to prevent the agent from pursuing outside rewards too aggressively. Thus, it may be possible to maintain unchanged incentives along this dimension with less costly labor rewards. The formal analysis combines elements of Grossman and Hart's (1983) two-step approach with techniques used in the first-order approach. See e.g. Rogerson (1985), Jewitt (1988), Conlon (2009), and Kirkegaard (2017). Only the latter allows a multi-dimensional action. The technical contribution of the paper is to extend these methods to settings with private rewards.

2 Model and preliminaries

2.1 The problem

The agent performs two "tasks", a_1 and a_2, each of which belong to a compact interval, $a_i \in [\underline{a}_i, \overline{a}_i]$, $i = 1, 2$. The first task captures the agent's effort on the job, as a result of which a contractible signal, x_1, is produced. The signal's marginal distribution is $G^1(x_1|a_1)$. The second "task" reflects the agent's pursuit of a private reward. The agent receives a possibly non-monetary reward, x_2, which is determined by the marginal distribution function $G^2(x_2|a_2)$. Assume x_i belongs to a compact interval, $[\underline{x}_i, \overline{x}_i]$, which is independent of a_i. Let $g^1(x_1|a_1)$ and $g^2(x_2|a_2)$ denote the densities and assume that $g^i(x_i|a_i) > 0$ for all $x_i \in [\underline{x}_i, \overline{x}_i]$ and all $a_i \in [\underline{a}_i, \overline{a}_i]$.[4] Note that each marginal distribution depends only on one task. This is further strengthened by assuming that x_1 and x_2 are independent.

Assumption A1 (Independence): *Outcomes are independent*, i.e. given a_1 and a_2, the joint distribution is given by

$$F(x_1, x_2|a_1, a_2) = G^1(x_1|a_1)G^2(x_2|a_2). \tag{1}$$

Independence implies that results are driven solely by the interactions in the agent's utility function. There are no confounding effects from a motive to manipulate the dependence structure. Moreover, there is no reason to think that the quality of the foliage on the agent's hike is correlated with how productive he is on the job. Likewise, the agent's performance on the job is unlikely to influence how many riders he gets on the weekend when he drives for Uber.

A contract is a function $w(x_1)$ that specifies the wage paid to the agent for any signal realization. If the agent takes action (a_1, a_2) and the rewards on and outside the job are w and x_2, respectively, then the agent's Bernoulli utility is

$$v(w, x_2) - c(a_1, a_2), \tag{2}$$

where v is a rewards function and c is a cost function. Thus, given the contract

[4] Throughout, all exogenous functions are assumed continously differentiable to the requisite degree. For brevity, statements to that effect are omitted from the numbered assumptions.

$w(x_1)$, the agent's expected payoff from action (a_1, a_2) is

$$EU(a_1, a_2|w(\cdot)) = \int\int v(w(x_1), x_2) g^1(x_1|a_1) g^2(x_2|a_2) dx_1 dx_2 - c(a_1, a_2). \quad (3)$$

The principal is risk neutral. Let $B(a_1, a_2)$ denote his benefit of the agent's action and assume that it is continuously differentiable. A leading case is when $B(a_1, a_2)$ depends only on a_1, e.g. when it is the expected value of x_1 given a_1. Then, a_2 is of interest to the principal only insofar as it can be manipulated to minimize the cost of implementing effort on the job. More generally, however, the principal may be directly interested in a_2. Finally, let $E[w|a_1, a_2]$ denote expected wage costs if the agent is induced to take action (a_1, a_2).

Assumption P1 (The principal's preferences): The principal is risk neutral, with expected utility $B(a_1, a_2) - E[w|a_1, a_2]$.

The principal's problem is to maximize his expected utility, subject to the participation constraint (P) and the incentive compatibility constraint (IC), or

$$\max_{a_1, a_2, w} B(a_1, a_2) - \int w(x_1) g^1(x_1|a_1) dx_1$$

$$st. \quad EU(a_1, a_2|w(\cdot)) \geq \overline{u} \quad \text{(P)}$$

$$(a_1, a_2) \in \arg \max_{(a_1', a_2') \in [\underline{a}_1, \overline{a}_1] \times [\underline{a}_2, \overline{a}_2]} EU(a_1', a_2'|w(\cdot)), \quad \text{(IC)}$$

where $\overline{u}$ is the agent's reservation utility. Any action that solves the problem is referred to as a second-best action.

The problems faced by the two parties have now been outlined in broad terms. However, more specific assumptions are required to solve these problems and generate economic insights. Thus, to continue, define $l^i(x_i|a_i) = \ln g^i(x_i|a_i)$. Let $l^i_{a_i}(x_i|a_i)$ denote the likelihood-ratio, i.e. the derivative of $l^i(x_i|a_i)$ with respect to a_i, $i = 1, 2$, and assume it is bounded. The next assumption is standard.

Assumption A2 (MLRP): The marginal distributions have the (strict) *monotone likelihood ratio property*, i.e. for all $a_i \in [\underline{a}_i, \overline{a}_i]$ it holds that

$$\frac{\partial}{\partial x_i}\left(l^i_{a_i}(x_i|a_i)\right) = \frac{\partial^2 \ln g^i(x_i|a_i)}{\partial a_i \partial x_i} > 0 \text{ for all } x_i \in [\underline{x}_i, \overline{x}_i]. \quad (4)$$

Rogerson (1985) considers a one-signal, one-task model and assumes that the distribution function is convex in the one-dimensional action. Kirkegaard (2017) allows multiple tasks and signals and assumes that the distribution function is convex in the many-dimensional action. The same assumption is useful here.

Assumption A3 (LOCC): $F(x_1, x_2|a_1, a_2)$ satisfies the *lower orthant convexity condition*; $F(x_1, x_2|a_1, a_2)$ is weakly convex in (a_1, a_2) for all (x_1, x_2) and (a_1, a_2).

Assumptions A1–A3 describe the agent's "technology". His preferences are described by Bernoulli utility of the form in (2). The rewards function $v(w, x_2)$ is strictly increasing and strictly concave in each argument, $v_i > 0 > v_{ii}$, $i = 1, 2$, where subscripts denote derivatives.[5] The domain is $\mathbb{R} \times [\underline{x}_2, \overline{x}_2]$. The assumption that $w \in \mathbb{R}$ ensures that wages are interior. Thus, wages may be negative.

The paper focuses on the case where rewards and tasks are substitutes. Thus, assume that $v_{12} < 0$; the higher x_2 is, the lower is the marginal utility of additional employment income. It should be acknowledged that rewards or actions are sometimes complements in the real world. For instance, it is perhaps the case that additional income is better enjoyed if the agent is in good health rather than in bad health. The model does not cover such situations.

Likewise, a_1 and a_2 are weak substitutes in the cost function, or $c_{12} \geq 0$. That is, the marginal cost of increasing a_1 is higher when a_2 is high. This assumption can be relaxed more easily, however, as discussed in Section 4. The assumption is satisfied in e.g. Holmström and Milgrom's (1991) leading model, the effort and attention allocation model, where costs depend only on $a_1 + a_2$. The cost function is strictly increasing and jointly convex in (a_1, a_2).

Assumption A4 (Monotonicity/substitutes): The agent's Bernoulli utility is $v(w, x_2) - c(a_1, a_2)$; $v(w, x_2)$ is strictly increasing and strictly concave in each argument separately, with domain $\mathbb{R} \times [\underline{x}_2, \overline{x}_2]$, while $c(a_1, a_2)$ is strictly increasing and weakly convex in (a_1, a_2). Rewards are strict substitutes; $v_{12}(w, x_2) < 0$. Tasks are weak substitutes; $c_{12}(a_1, a_2) \geq 0$.

A detailed discussion of assumptions can be found in the Online Appendix.

[5]It is not necessary for $v(w, x_2)$ to be jointly concave in (w, x_2) in order for the agent's expected utility to be concave. See Lemma 1, below, and its proof. For comparison, in Rogerson (1985) concavity of the Bernoulli utility function is used only to prove that the contract is monotonic. Beyond that, it is not invoked to prove that the agent's problem is concave.

2.2 Preliminary observations

At this stage, the properties of the endogenous contract are unknown. To get a feel for the problem, however, it is useful to begin by considering the agent's problem if the contract $w(x_1)$ is differentiable and strictly increasing.

Definition (regular contracts): The contract is said to be *regular* if it is differentiable and strictly increasing, with $w'(x_1) > 0$ for all $x_1 \in [\underline{x}_1, \overline{x}_1]$.

Now, $EU(a_1, a_2|w(\cdot))$ is strictly concave in the agent's action when the contract is regular. In this case, the agent's optimal action is unique.

Lemma 1 *Assume that A1–A4 hold. For any regular contract, EU is strictly concave in* (a_1, a_2)*, with* $EU_{11} < 0$*,* $EU_{22} < 0$*,and* $EU_{11}EU_{22} - EU_{12}^2 > 0$*. Moreover, the two tasks are strict substitutes, or* $EU_{12} < 0$*.*

Proof. See the Appendix. ■

To compare the agent's problem to a more standard problem, define

$$V(w, a_2) = \int v(w, x_2) g^2(x_2|a_2) dx_2 \tag{5}$$

as the expected utility of a fixed wage, given that the agent exerts effort a_2 towards private rewards. Note that the expectation is over x_2, given a_2. Then,

$$EU(a_1, a_2|w(\cdot)) = \int V(w(x_1), a_2) g^1(x_1|a_1) dx_1 - c(a_1, a_2). \tag{6}$$

If a_2 is exogenous, the expression in (6) is of course identical to expected utility in a standard model where Bernoulli utility and costs are parameterized by a_2. In this case, it is well known that the participation constraint must bind if the contract $w(x_1)$ is optimal. Otherwise, construct another contract, $\widehat{w}_\varepsilon(x_1)$, such that $V(\widehat{w}_\varepsilon(x_1), a_2) = V(w(x_1), a_2) - \varepsilon$ for all x_1, with $\varepsilon > 0$. The two contracts induce the same a_1 and both satisfy (P) if ε is small enough. Since $\widehat{w}_\varepsilon(x_1)$ entails lower wages, $w(x_1)$ cannot be optimal for the principal. Thus, the standard model where leisure is treated as a black box predicts that the agent earns no more and no less than reservation utility. For future reference, implementation costs are strictly increasing in a_1 when a_2 is exogenous, see e.g. Jewitt et al (2008).

However, the contracting problem is more complex when a_2 is endogenous. Changing the contract from $w(x_1)$ to $\widehat{w}_\varepsilon(x_1)$ induces the agent to reconsider his *joint* choice of a_1 and a_2. Intuitively, the job becomes a worse source of rewards and so the agent is likely to shift his attention towards pursuing private rewards. However, as private rewards increase, the incentive to pursue labor rewards lessens. Thus, a_2 increases and a_1 decreases as a result.

Proposition 1 *Assume that A1–A4 hold. Fix a regular contract, $w(x_1)$, and let (a_1^*, a_2^*) denote the interior action that it induces. If the agent participates, let (a_1', a_2') denote the action that is induced when $w(x_1)$ is replaced by $\widehat{w}_\varepsilon(x_1)$ where $V(\widehat{w}_\varepsilon(x_1), a_2^*) = V(w(x_1), a_2^*) - \varepsilon$, $\varepsilon > 0$. Then, $a_1' < a_1^*$ and $a_2' > a_2^*$.*

Proof. See the Appendix. ■

Proposition 1 establishes that the standard argument used to show that (P) binds when a_2 is exogenous breaks down when a_2 is endogenous. Indeed, a key message of the paper is that (P) may not bind when a_2 is endogenous. Moreover, even if the principal does not care directly about a_2, he must still carefully consider which level of a_2 to induce. Proposition 1 suggests that a lower level of a_2 makes it easier to induce effort on the job. On the other hand, the agent is willing to lower a_2 only if the job is a good source of utility. The first effect appears to be beneficial to the principal but the second effect appears to be detrimental. Thus, there is a trade-off when determining which a_2 to induce.[6]

Conversely, imagine inducing higher effort on the job, a_1, while keeping effort in pursuit of private rewards, a_2, constant. Intuitively, there are two competing effects. First, steeper incentives are seemingly required to entice higher effort on the job. On the other hand, since a_1 and a_2 are substitutes, it becomes easier to induce the agent to maintain an unchanged level of a_2. Thus, it may be possible to lower wages for at least some values of x_1. In other words, the cost of incentivizing a_1 increases while the cost of incentivizing a_2 decreases. Hence, there is a trade-off. In contrast to the standard model, one should therefore not necessarily expect implementation costs to be monotonic in a_1, holding a_2 fixed.

[6] Note that these complications disappear entirely if V is additively separable, or $V_{12} = 0$. Then, the standard argument applies and (P) binds. Moreover, for any given a_1 there is a unique implementable a_2. The LEN model should be seen in this light, since in that model the agent's certainty equivalent is additively separable in rewards.

2.3 Towards optimal contracts

Following Grossman and Hart (1983), a two-step procedure is sometimes used to derive the second-best action. In the first step, the optimal contract that induces any fixed action is derived and implementation costs calculated. The second step then maximizes over the action to identify the second-best action.

For expositional simplicity, the paper mainly focuses on interior actions. For any interior action, incentive compatibility necessitates that expected utility achieves a stationary point at that action, or $EU_1 = 0 = EU_2$. These constraints are referred to as the "local" incentive compatibility constraints. The shorthand L-IC$_i$ is used to refer to the constraint that $EU_i = 0$, $i = 1, 2$, while L-IC refers to L-IC$_1$ and L-IC$_2$ together. To find the optimal contract that induces a given action, one can minimize implementation costs subject to (P) and L-IC. Of course, this necessitates that the constraint set is non-empty, i.e. there is a contract satisfying (P) and L-IC. Assume this is the case for now. If the resulting candidate solution is a regular contract, then the agent's problem is concave and L-IC is indeed sufficient for incentive compatibility. A key technical challenge is thus to establish that the candidate solution is regular.

Thus, holding fixed the action, consider the solution to the above problem. Let $\lambda \geq 0$ denote the multiplier on the participation constraint. Let μ_1 and μ_2 denote the multipliers on L-IC$_1$ and L-IC$_2$, respectively. The optimal wage if x_1 is observed is implicitly characterized by the necessary first order condition

$$\lambda + \mu_1 l^1_{a_1}(x_1|a_1) = \frac{1}{V_1(w(x_1), a_2)} - \mu_2 \frac{V_{12}(w(x_1), a_2)}{V_1(w(x_1), a_2)}. \tag{7}$$

The last term in (7) generally complicates the analysis. To begin, a special model specification in which this complication is minimized is considered.

Definition (The multiplicative model): In the *multiplicative model*, the rewards function is

$$v(w, x_2) = -m(w)n(x_2), \tag{8}$$

where m and n are strictly *negative* functions that are strictly increasing and strictly concave.

Rewards are substitutes in the multiplicative model.[7] For example, imagine private rewards are monetary and that the agent has constant absolute risk aversion over total income. Then, utility from rewards is $v(w, x_2) = -e^{-r(w+x_2)}$, $r > 0$. This fits the multiplicative model, with $m(w) = -e^{-rw}$ and $n(x_2) = -e^{-rx_2}$.

A distinguishing feature of the multiplicative model is that the last term in (7) is independent of w. Then, holding fixed the action, (7) can be rewritten as

$$\frac{1}{V_1(w(x_1), a_2)} = \widehat{\lambda} + \mu_1 l^1_{a_1}(x_1|a_1). \tag{9}$$

In this case, (9) takes the same form as in the standard model. Thus, standard arguments apply. It follows that $\mu_1 > 0$ and, given MLRP, that the contract is regular; See Rogerson (1985, footnote 8) or Jewitt (1988, Lemma 1). By Lemma 1, L-IC is then sufficient for incentive compatibility. This confirms that the optimal contract that induces the interior action in question – provided it is implementable in the first place – does indeed take the form in (7) or (9).

The two binding constraints, L-IC$_1$ and L-IC$_2$, can be used to solve for the two unknowns, $\widehat{\lambda}$ and μ_1, in (9). Note that (P) plays no role whatsoever in deriving this candidate contract. Thus, it would be purely coincidental if (P) happens to bind. It is entirely possible that the agent earns more than reservation utility. On the other hand, it is also possible that the candidate contract violates (P). In this case, it will be shown later that the action is not implementable.

The following example illustrates several of the paper's main findings.

Example 1: Assume that $a_i \in \left[\frac{1}{9}, \frac{1}{8}\right]$, $i = 1, 2$, and that

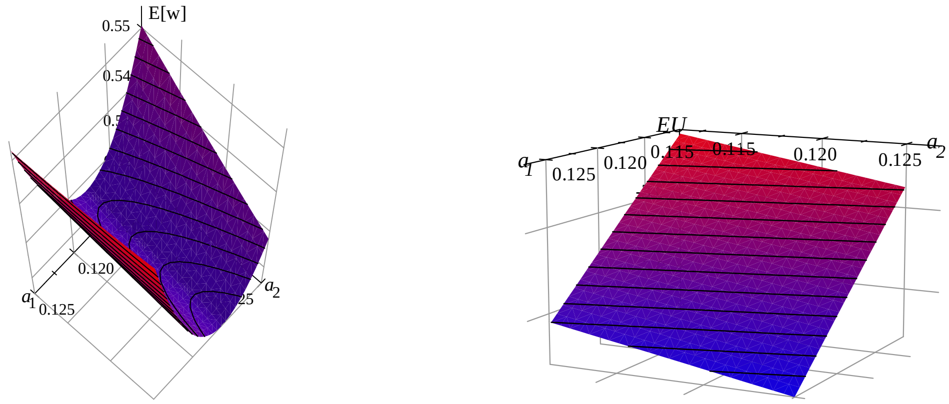
$$G^i(x_i|a_i) = \left(1 - e^{-a_i}\right) x_i^2 + \left(e^{-a_i}\right) x_i, \quad x_i \in [0, 1].$$

It can be verified that Assumptions A1–A3 are satisfied. Assume also that

$$c(a_1, a_2) = 32a_1^2 - 8a_1 + \frac{1}{2}a_2^2 + \frac{3}{40}a_2 + 8a_1a_2,$$

which is strictly increasing and convex and satisfies $c_{12} > 0$.

[7] The term "multiplicative" may invoke thoughts of complementarity rather than substitutability. However, note that the product of the two (negative) functions is multiplied by -1. For this reason, w_1 and x_2 are substitutes.



(a) Implementation costs.

(b) Expected utility.

Figure 1: Costs and utility from implementing different actions.

Finally, the rewards function is multiplicative, with

$$v(w, x_2) = -\left(2\sqrt{w} - k\right)\left(7.5\sqrt{x_2} - 8\right),$$

where $w \in [0, (\frac{k}{2})^2]$, $x_2 \in [0, 1]$ and $k = 2.685$. Assumption A4 is satisfied, with the exception that w cannot be any real number. However, k is selected to ensure that any optimal contract features wages in the interior and that $m(w) = 2\sqrt{w} - k$ is negative as required in the multiplicative model. It can be shown that

$$V(w, a_2) = \left(2\sqrt{w} - k\right)\left(e^{-a_2} + 2\right).$$

Rewards functions with square-root utility have been used in standard models to derive optimal contracts. See e.g. Jewitt et al (2008) or Kirkegaard (2017). Similar techniques can be used here. As indicated above, L-IC$_1$ and L-IC$_2$ are used to derive the multipliers. Thus, the contract can be characterized in closed form and implementation costs can be derived. Details are available on request.

Implementation costs are illustrated in Figure 1(a) for interior actions, assuming all such actions are implementable. This is the case if and only if reservation utility, $\overline{u}$, is low enough. Figure 1(b) depicts the agent's expected utility from the optimal contract that induces any interior action. Note that the agent is worse off the closer the induced action is to $(a_1, a_2) = \left(\frac{1}{8}, \frac{1}{8}\right)$. Thus, as $\overline{u}$ increases, actions near $(a_1, a_2) = \left(\frac{1}{8}, \frac{1}{8}\right)$ are no longer implementable. In Figure 1, the curves on the surfaces are iso-cost curves and indifference curves, respectively.

The example exhibits a number of interesting properties.

Locally increasing or decreasing implementation costs: Figure 1(a) shows that implementation costs are U-shaped in a_1 when a_2 is held fixed. In particular, there is a range where costs decrease as higher a_1 is induced. Implementation costs are decreasing in a_2 as long as a_1 is not too large. However, this is not true when a_1 is large. For instance, the cheapest interior a_2 to induce alongside $a_1 = 0.124$ is $a_2 = 0.118$ (all values are rounded to the third decimal place). Implementation costs increase as a_2 moves away from this value in either direction. These results prove that the trade-offs discussed after Proposition 1 may go in either direction.

Slack participation constraint: Imagine that the principal wants to induce $a_1 = 0.124$ and that he does not care directly about a_2. Hence, he seeks to induce whichever a_2 minimizes implementation costs along with $a_1 = 0.124$. If reservation utility is $\overline{u} = -3.527$, say, then $a_2 = 0.118$ is implementable and cost minimizing, given $a_1 = 0.124$.[8] The agent's expected utility is $-3.484 > -3.527$.

Weaker incentives: In this example, it can be verified that the optimal contracts that induce the same a_1 but different a_2 cross each other exactly once. The optimal contract that induces the lower a_2 crosses the other one from above and thus features wages that are more "compressed." For example, assuming that $\overline{u} = -3.527$, (P) binds under the optimal contract that induces $(a_1, a_2) = (0.124, 0.124)$. This contract is depicted as the dashed curve in Figure 2 alongside the optimal contract that induces $(a_1, a_2) = (0.124, 0.118)$ (the unbroken curve). Intuitively, the more compressed contract carries less risk for the agent and is therefore more attractive. Since work is now a good source of utility, the agent relaxes his pursuit of outside rewards. Importantly, in this particular case the more compressed contract features a lower expected wage as well.

Incentives, utility, and work-life balance: Summarizing the above findings, the contract that minimizes the cost of implementing $a_1 = 0.124$ skews the agent's

[8] At the same time, reservation utility is low enough that $a_2 = \overline{a}_2$ cannot be implemented. In this example, it is always cheapest to implement $\overline{a}_2$ whenever feasible.

work-life balance away from life ($a_2 = 0.118$ instead of $a_2 = 0.124$) compared to the contract that forces (P) to bind. At the same time, the agent faces weaker incentives and earns higher expected utility.[9] ▲

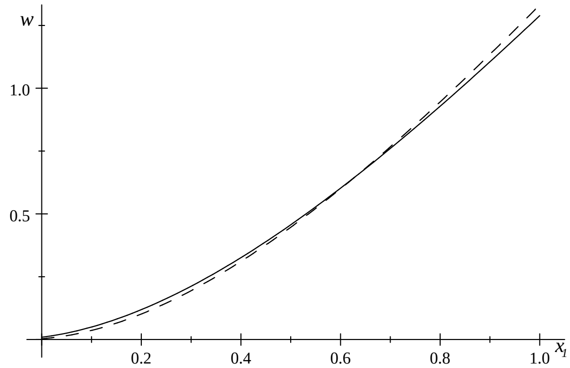


Figure 2: Comparing contracts.

3 A reduced problem

Recall that the optimal contract that induces any given interior and implementable action in the multiplicative model is derived from L-IC. The only role played by the participation constraint is to determine whether the action is implementable at all. This section generalizes this result beyond the multiplicative model and pursues the economic implications in a series of steps, as follows.

First, an important new assumption with a compelling economic interpretation is introduced. The multiplicative model is a special case and (7) is still tractable. Second, it is shown that there is a set of interior actions for which the participation constraint is redundant. Third, a consequence is that any action in this set is optimally implemented with a regular contract that takes the form in (7). Note that these findings extend those in Example 1 beyond the multiplicative model. Recall that Example 1 also illustrates that implementation costs may be increasing or decreasing in a_1 or a_2. The fourth contribution of this section is to generalize this finding and to explain more generally under what circumstances costs are increasing or decreasing in a_1 or a_2.

[9]The contract can also be compared to the one that is optimal if $a_2 = 0.118$ is exogenous. Under exogenous a_2, (P) binds and the wage in this example simply decreases such that $v(w(x_1), x_2)$ and $V(w(x_1), a_2)$ shift down by a constant. This does not generalize, however.

Fifth, the set of actions for which (P) is redundant has the agent working less hard on outside activities than he would under the cost minimizing contract under symmetric information. Thus, the agent's work-life balance is shifted away from "life" compared to the symmetric-information benchmark. Hence, he must be compensated with utility in excess of reservation utility if his work-life balance is to be distorted away from "life." Finally, it is argued that incentives are in some sense flatter in this case too. Thus, the optimal contract exhibits properties that are similar to those in Englmaier and Leider's (2012) reciprocity-based model.

Generally speaking, actions for which (P) is not redundant may also be implementable, although this is not the case in the multiplicative model. For completeness, this possibility is dealt with in Section 4.

3.1 Aggregation

A final assumption is imposed on the agent's rewards function.

Assumption A5 (Log-supermodularity): The agent's marginal utility of labor income, $v_1(w, x_2)$, is log-supermodular in (w, x_2), or

$$\frac{\partial^2 \ln v_1(w, x_2)}{\partial w \partial x_2} \geq 0 \text{ for all } w \text{ and all } x_2 \in [\underline{x}_2, \overline{x}_2]. \tag{10}$$

Assumption A5 is equivalent to assuming that

$$\frac{\partial}{\partial x_2}\left(\frac{-v_{11}(w, x_2)}{v_1(w, x_2)}\right) \leq 0 \text{ for all } w \text{ and all } x_2 \in [\underline{x}_2, \overline{x}_2]. \tag{11}$$

Thus, the agent's absolute risk aversion over labor income is decreasing in x_2. In other words, the agent is less sensitive to risk in labor income when the private reward is high. If the private reward is monetary and $v(w, x_2)$ takes the form $u(w+x_2)$, then Assumption A5 is equivalent to the assumption that the agent has decreasing absolute risk aversion with respect to total income. In the multiplicative model, the inequality in (10) is an equality. Assumption A5 is particularly important because it helps impose structure on the aggregate function $V(w, a_2)$.

Lemma 2 *Given A1–A5, the function $V(w, a_2)$ has the following properties:*

1. $V(w, a_2)$ is strictly increasing and strictly concave in both arguments,

$$V_i(w, a_2) > 0 > V_{ii}(w, a_2), \quad i = 1, 2.$$

2. w and a_2 are strict substitutes, $V_{12}(w, a_2) < 0$.

3. $V_1(w, a_2)$ is log-supermodular in (w, a_2),

$$\frac{\partial}{\partial a_2}\left(\frac{-V_{11}(w, a_2)}{V_1(w, a_2)}\right) = \frac{\partial}{\partial w}\left(\frac{-V_{12}(w, a_2)}{V_1(w, a_2)}\right) \leq 0. \tag{12}$$

4. Parts 1–3 imply that $V_{112}(w, a_2) > 0$.

Proof. See the Appendix. ■

The last part of (12) imposes useful structure on the last term in (7). Now, the properties of the contract that solves (7) depend on the signs of the two multipliers μ_1 and μ_2, which at this point are unknown. However, the contract $w(x_1)$ is regular if it turns out to be the case that $\mu_1 > 0 \geq \mu_2$. Technically, then, the challenge becomes to verify that $\mu_1 > 0 \geq \mu_2$.

Lemma 3 *Given A1-A5 and P1, $w(x_1)$ as defined in (7) is unique whenever $\mu_1 \geq 0 \geq \mu_2$. Moreover, $w(x_1)$ is regular if $\mu_1 > 0 \geq \mu_2$.*

Proof. See the Appendix. ■

3.2 Redundant participation constraint

Note that L-IC$_2$ can be written as

$$\int \left[-V_2(w(x_1), a_2)\right] g^1(x_1|a_1)dx_1 = -c_2(a_1, a_2).$$

The function $[-V_2(w, a_2)]$ is strictly increasing and strictly concave in w, since $V_{12} < 0$ and $V_{112} > 0$. Thus, the idea is to think of it as an artificial utility function. The properties in Lemma 2 imply that the artificial utility function is weakly more risk averse than the real utility function, $V(w, a_2)$. Thus, the former can be used to bound the latter. The multiplicative model captures the special case where the two utility functions are equally risk averse.

To begin, note that L-IC$_2$ nails down the certainty equivalent, as evaluated by the artificial utility function, of any contract that induces a_2. Let this certainty equivalent be denoted $CE_{-V_2}(a_1, a_2)$, with

$$-V_2(CE_{-V_2}(a_1, a_2), a_2) = -c_2(a_1, a_2). \tag{13}$$

For expositional simplicity, assume throughout that $CE_{-V_2}(a_1, a_2)$ exists. Otherwise, (a_1, a_2) has no chance of being implementable. A sufficient condition is that $v_2(w, x_2) \to 0$, and thus $V_2(w, a_2) \to 0$, as $w \to \infty$.

Next, expected utility from a fixed-wage contract paying $CE_{-V_2}(a_1, a_2)$ is

$$\overline{U}(a_1, a_2) \equiv V(CE_{-V_2}(a_1, a_2), a_2) - c(a_1, a_2), \tag{14}$$

when the agent takes action (a_1, a_2). Let

$$T(\overline{u}) = \{(a_1, a_2) \in [\underline{a}_1, \overline{a}_1] \times [\underline{a}_2, \overline{a}_2] | \overline{U}(a_1, a_2) \geq \overline{u}\}$$

describe the set of actions for which the agent would achieve at least reservation utility by taking action (a_1, a_2) given a fixed-wage contract paying $CE_{-V_2}(a_1, a_2)$. Clearly, this ignores incentive compatibility with respect to a_1.

To reiterate, all contracts that implement (a_1, a_2) share the same $CE_{-V_2}(a_1, a_2)$ value as determined from L-IC$_2$. Since the certainty equivalent of any contract is weakly larger when evaluated with $V(w, a_2)$ than with $[-V_2(w, a_2)]$, it holds that

$$\begin{aligned} EU(a_1, a_2|w(\cdot)) &= \int V(w(x_1), a_2) g^1(x_1|a_1) dx_1 - c(a_1, a_2) \\ &\geq V(CE_{-V_2}(a_1, a_2), a_2) - c(a_1, a_2) = \overline{U}(a_1, a_2). \end{aligned} \tag{15}$$

Since $CE_{-V_2}(a_1, a_2)$ is predetermined, $\overline{U}(a_1, a_2)$ is also predetermined. Holding fixed the action, the agent's expected utility thus exceeds $\overline{u}$ if $\overline{u}$ is small enough, $\overline{u} < \overline{U}(a_1, a_2)$. Then, the participation constraint is automatically satisfied and slack. Stated differently, if $\overline{u}$ is small then (P) is redundant – it is implied by L-IC$_2$ – and the agent is guaranteed rent strictly above his reservation utility.

Proposition 2 *Assume A1–A5 hold and fix any interior action that is to be induced. Then, the participation constraint is redundant if $\overline{u}$ is small enough. Conversely, if $\overline{u}$ is held fixed then the participation constraint is redundant for any interior action in $T(\overline{u})$.*[10]

Proof. The first part is in the text. The second part is by definition of $T(\overline{u})$. ■

Laffont and Martimort (2002, Section 5.3) present a single-task model without private rewards. However, their utility function is not separable in income and effort. Although (P) is not redundant in their model, it may still be slack.[11] In a standard separable model, an increase in the agent's base wage would necessitate an increase in performance pay in order to restore incentives. In Laffont and Martimort (2002), however, the agent is more risk averse when he works harder. The increase in base wage may then be more valuable if the agent works hard than if he does not. Thus, it may be possible to lower performance pay and still maintain incentives. This yields a more compressed wage schedule, which may be to both the agent's and the principal's advantage. Hence, there are some similarities between the models' conclusions. In the current model, a change in the contract changes a_2, which changes $V(w, a_2)$ and $c(a_1, a_2)$. This indirectly creates a type of non-separability between the contract and effort on the job, a_1.

3.3 The reduced problem

Proposition 2 implies that the principal can ignore the participation constraint if he is seeking to induce any interior action in $T(\overline{u})$. Thus, to minimize implementation costs for any fixed interior action in $T(\overline{u})$, consider the *reduced problem*

$$\max_{w} - \int w(x_1) g_1(x_1|a_1) dx_1$$
$$st. \quad EU_i(a_1, a_2|w(\cdot)) = 0, \quad i = 1, 2. \tag{L-IC}$$

[10] This result extends to actions $(a_1, \underline{a}_2)$ when a_1 is interior and $(a_1, \underline{a}_2) \in T(\overline{u})$. Then, L-IC$_2$ must be replaced by $EU_2 \leq 0$, or $V_2(CE_{-V_2}(a_1, \underline{a}_2) \leq c_2(a_1, \underline{a}_2)$. Since $V_{12} < 0$, this holds only if CE_{-V_2} is as defined in (13), or larger. The rest of the argument is then unchanged.

[11] The agent has only two actions available to him in Laffont and Martimort's (2002) model. Alvi (1997) justifies the first-order approach in a fairly similar model but with a continuous action. He does not discuss the participation constraint in any detail, but he does note that non-separability tends to make the contract flatter.

It can be established that any contract that solves the reduced problem is regular. The structure of the contract is as in (7), with $\lambda = 0$ and $\mu_1 > 0 > \mu_2$. Hence, the contract is incentive compatible, by Lemmata 1 and 3.

Theorem 1 *Assume that A1–A5 and P1 holds. The optimal contract that induces any interior action in $T(\overline{u})$ solves the reduced problem and takes the form in (7), with $\lambda = 0$ and $\mu_1 > 0 > \mu_2$.*

Proof. See the Appendix. ■

Keeping Grossman and Hart's (1983) two-step procedure in mind, Theorem 1 makes it possible to calculate implementation costs for any interior action in $T(\overline{u})$. Note that if $\overline{u}$ is small enough then $T(\overline{u})$ is of course the entire set of actions. Moreover, the reduced problem gives a lower bound on implementation costs for interior actions that are outside $T(\overline{u})$. Thus, one can solve the reduced problem for all interior actions and then use this to derive a candidate for the second-best action. If this is in the interior of $T(\overline{u})$, then the correct solution has been obtained. Section 4 outlines a solution method that rigorously solves the problem without assuming that the second-best is in $T(\overline{u})$.

Example 1 shows that implementation costs may be locally increasing or decreasing in a_1 and a_2. This is not specific to the example but is rather a general property, in the following sense. Holding fixed the agent's reward function and reservation utility, any given interior action is in $T(\overline{u})$ if $c(a_1, a_2)$ is small enough. Then, the optimal contract that solves the reduced problem relies on c_1 and c_2 but is independent of any other features of the cost function, like c_{11}, c_{22}, or c_{12}. By the Envelope Theorem, however, the latter features affect how implementation costs change with a marginal change in a_1 and a_2. It turns out that it is always possible to find a cost function that satisfies Assumption A4 and for which implementation costs are locally increasing or locally decreasing in a_1 or a_2.

Proposition 3 *Fix an interior action (a_1^*, a_2^*) and fix $(c(a_1^*, a_2^*), c_1(a_1^*, a_2^*), c_2(a_1^*, a_2^*))$ in such a way that $(a_1^*, a_2^*) \in T(\overline{u})$. Then, there exists cost functions that satisfy Assumption A4 and for which:*

1. *$E[w|a_1, a_2]$ is locally increasing (decreasing) in a_1 at (a_1^*, a_2^*) if $c_{11}(a_1^*, a_2^*) > 0$ is large (small) relative to $c_{22}(a_1^*, a_2^*)$ and $c_{12}(a_1^*, a_2^*)$.*

2. *$E[w|a_1, a_2]$ is locally increasing (decreasing) in a_2 at (a_1^*, a_2^*) if $c_{22}(a_1^*, a_2^*) > 0$ is small (large) relative to $c_{11}(a_1^*, a_2^*)$ and $c_{12}(a_1^*, a_2^*)$.*

Proof. See the Appendix ■

To explain the first part of Proposition 3, recall that there are two competing effects from incentivizing higher effort on the job. First, the marginal cost of effort on the job, c_1, changes, with c_{11} measuring the size of the change. Likewise, the marginal cost of effort in pursuit of private rewards, c_2, changes, with c_{12} capturing the speed of the change. When c_2 changes quickly (c_{12} is large), the agent who works harder on the job is much less inclined to work hard outside the office. Thus, L-IC$_2$ becomes substantially cheaper to satisfy. If c_1 changes slowly at the same time (c_{11} is small), then the cost of satisfying L-IC$_1$ only increases slightly. Then, the combined costs of L-IC$_1$ and L-IC$_2$ fall.[12] In this case, it is cheaper to induce marginally higher effort on the job. Similar intuition, which is taken up in the next subsection, explains the second part of the proposition.[13]

3.4 The reduced problem and work-life balance

Recall that the participation constraint is slack for any action in the interior of $T(\overline{u})$. Thus, it is of interest to examine the properties of this set. First, it can be shown that $\overline{U}(a_1, a_2)$ is strictly decreasing in both arguments. This is turn implies that $T(\overline{u})$ is a "decreasing set"; if some action is in the set then all smaller actions are also in the set. Likewise, the set grows bigger when $\overline{u}$ declines.

Proposition 4 *Given A1–A5, $\overline{U}(a_1, a_2)$ is strictly decreasing in a_1 and a_2. Moreover, $T(\overline{u})$ is a decreasing set: If $(a_1, a_2) \geq (a_1', a_2')$ and $(a_1, a_2) \in T(\overline{u})$ then $(a_1', a_2') \in T(\overline{u})$. Finally, if $\overline{u} > \overline{u}'$ then $T(\overline{u}) \subseteq T(\overline{u}')$.*

Proof. See the Appendix. ■

[12] In the statement of Proposition 3, c_{11} must be small not only relative to c_{12} but also relative to c_{22}. This is to ensure that the convexity condition $c_{11}c_{22} - c_{12}^2 \geq 0$ is satisfied as c_{12} increases. Incidentally, the cost function in Example 1 satisfies $c_{11}c_{22} - c_{12}^2 = 0$.

[13] Care should be taken when comparing Proposition 3 and Example 1. The former is a local comparative statics result that fixes an action but allows different cost functions to be compared. Example 1 fixes a specific cost function but allows global changes in the action.

To relate Proposition 4 to Example 1, note that the inequalities in (11) and (12) hold with equality in the multiplicative model. Consequently, $V(w, a_2)$ and $-V_2(w, a_2)$ are equally risk averse and have the same certainty equivalent. Hence, $EU(a_1, a_2|w(\cdot)) = \overline{U}(a_1, a_2)$; (15) holds as an equality. As evidenced by Figure 1(b), $\overline{U}(a_1, a_2)$ is indeed strictly decreasing in both arguments. An important implication of $EU(a_1, a_2|w(\cdot)) = \overline{U}(a_1, a_2)$ is that any interior action that is outside $T(\overline{u})$ and which satisfies L-IC$_2$ cannot satisfy (P). Hence, such an action cannot be implemented. This argument extends to actions with $a_2 = \overline{a}_2$.

Proposition 5 *In the multiplicative model, any action with $a_1 > \underline{a}_1$ and $a_2 > \underline{a}_2$ can be implemented only if (a_1, a_2) it is in $T(\overline{u})$.*

Proof. See the Appendix. ■

Beyond the multiplicative model, Proposition 4 implies that actions involving "small" effort levels on one or both activities can only be incentivized by giving the agent more than his reservation utility. It turns out to be fruitful to make more precise in what sense the effort levels are small. In the spirit of Grossman and Hart (1983), begin by fixing some a_1 that is to be implemented. A range of a_2 levels can be implemented alongside a_1. Define

$$t(a_1) = \max\{a_2|(a_1, a_2) \in T(\overline{u})\}$$

such that $(a_1, t(a_1))$ describes the upper boundary of the set $T(\overline{u})$. Stated differently, the action (a_1, a_2) is in $T(\overline{u})$ if and only if $a_2 \leq t(a_1)$. The agent earns utility strictly above $\overline{u}$ if $a_2 < t(a_1)$.

Next, consider the following benchmarks. First, assume that there is no asymmetric information and thus no moral hazard. That is, the principal can dictate a_1 and a_2. Holding fixed a_1, it is natural to ask which a_2 level it should be accompanied with in order to minimize wage costs. Let $s(a_1)$ denote the a_2 level that solves this symmetric-information problem. Alternatively, let $p(a_1)$ denote the a_2 level that solves the partial-information problem of minimizing the cost of implementing a_1 when a_1 is contractible but a_2 and x_2 are not. Note that these benchmarks are most relevant when $B(a_1, a_2)$ is independent of a_2 such that the principal's choice of a_2 is motivated only by cost considerations. Here, one can think of the relative magnitudes of a_1 and $s(a_1)$ (or a_1 and $p(a_1)$) as reflecting the work-life balance that the agent would experience in the symmetric (or partial) information benchmark.

Proposition 6 *Fix an interior a_1 and assume that $t(a_1)$ is interior. Then, $s(a_1) = p(a_1) = t(a_1)$.*

Proof. See the Appendix. ■

Thus, actions in $T(\overline{u})$ entail lower levels of effort in pursuit of private rewards than under symmetric or partial information. In other words, the agent's work-life balance is skewed away from "life" or outside activities compared to the benchmarks. In these cases, the agent must be awarded with utility in excess of his reservation utility.

Corollary 1 *Fix an interior a_1 and assume that $t(a_1)$ is interior. Then, the agent must earn strictly more than reservation utility whenever his work-life balance is skewed further away from life than under symmetric or partial information, i.e. whenever $a_2 < s(a_1) = p(a_1) = t(a_1)$.*

It is worth remembering that the second part of Proposition 3 implies that implementation costs are locally increasing in a_2 around $t(a_1)$ if c_{12} and c_{11} are large relative to c_{22}. In this case, it is thus cheaper for the principal to skew the agent's work-life balance away from "life", even though the agent will have to be compensated with higher expected utility. Note that a large value of c_{12} – or a large degree of substitutability between a_1 and a_2 – implies that it becomes much easier to satisfy the L-IC$_1$ constraint when a_2 is lowered. At the same time, a small value of c_{22} means that the L-IC$_2$ constraint is impacted less by a decrease in a_2 and only becomes slightly harder to satisfy.

Finally, it should be clarified that outside the multiplicative model, actions for which $a_2 > t(a_1)$ may be implementable and may even leave (P) slack. In particular, imagine that the artificial utility function is strictly more risk averse than the real utility function. Since the optimal contract involves risk to satisfy L-IC$_1$, the former utility function then has a strictly smaller certainty equivalent than the latter. Thus, the inequality in (15) must be strict. Consequently, (P) is slack even if a_2 is slightly larger than $t(a_1)$. However, it is impossible to quantify exactly how large a_2 can be for this property to still hold.

3.5 Work-life balance and flatter incentives

Example 1 noted that the contract that induces a low level of a_2 is more compressed than the contract that induces the higher level of a_2 for which (P) binds. Given Corollary 1, this means that when the agent is induced to shift his work-life balance away from "life", he is manipulated to do so with a contract that has weaker incentives compared to the contract that would have maintained the symmetric-information level of work-life balance.

At a broad intuitive level this result carries over to the general model, with the caveat that "weaker incentives" is a poorly defined term and one that is hard to formalize in a setting where optimal contracts are typically non-linear. What is unambiguously true, however, is that the agent's marginal cost of effort on the job, $c_1(a_1, a_2)$, is lower and his marginal utility of labor income, as measured by $V_1(w, a_2)$, is higher when a_2 is small. Hence, the contract must be made flatter in some aggregate or average sense in order to restore incentives on the job when a smaller a_2 is induced. One possible way of formalizing this intuition follows.

The standard way of thinking of a contract is as a mapping from the signal, x_1, to the wage, w. However, this is not the only way to think about a contract. First, note that the optimal contract depends on the signal x_1 only through the likelihood-ratio, $l^1_{a_1}(x_1|a_1)$. Second, the utility function $V(\cdot, a_2)$ is used to evaluate the incentives for effort on the job for any given contract. Different utility functions gives different incentives, even for the same wage schedule. Combining these two observations leads to the idea of translating the contract into a mapping from the likelihood-ratio to the agent's utility. In other words, for any given realization of the likelihood-ratio, the contract gives the agent a certain amount of utils. In the current model, however, utility also depends on a_2.

Now fix two contracts, $\widehat{w}$ and $\widetilde{w}$, that optimally induce interior actions $(a_1, \widehat{a}_2)$ and $(a_1, \widetilde{a}_2)$, respectively. Assume $\widehat{a}_2 > \widetilde{a}_2$. It turns out that as long as the agent uses any utility function $V(\cdot, a_2)$ with $a_2 \in [\widetilde{a}_2, \widehat{a}_2]$ to evaluate incentives, there is a sense in which incentives for effort on the job are flatter with $\widetilde{w}$ than with $\widehat{w}$.

Proposition 7 *For any $a_2 \in [\widetilde{a}_2, \widehat{a}_2]$, the covariance between $V(\widetilde{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$ is strictly smaller than the covariance between $V(\widehat{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$.*

Proof. See the Appendix. ■

Proposition 7 implies that the slope of a regression of $V(\widetilde{w}(x_1), a_2)$ on $l^1_{a_1}(x_1|a_1)$ is strictly smaller than the slope of a regression of $V(\widehat{w}(x_1), a_2)$ on $l^1_{a_1}(x_1|a_1)$. The regression irons out the non-linearities in the agent's utility and thereby presents a way to think about "average" incentives. The slope can be thought of as the average piece-rate, measured in utils, for a marginal increase in the likelihood-ratio. Future research is planned to pursue other implications of this idea.

4 Discussion and extensions

This section briefly discusses the assumption that $c_{12} \geq 0$ and outlines a general solution method that does not restrict attention to actions in the set $T(\overline{u})$. The Online Appendix contains a more thorough discussion of boundary actions and of the assumptions of the model. A discussion of the model's relationship to the literatures on private investments and common agency can also be found there.

4.1 Complementary tasks

It has been assumed that tasks are substitutes, or $c_{12} \geq 0$. However, the arguments that led to the reduced problem do not depend on this assumption in any way. Hence, even if it is assumed that tasks are complements, or $c_{12} \leq 0$, there is a set $T(\overline{u})$ of actions on which the reduced problem identifies the optimal contract. Moreover, for a given a_1, there is once again a cut-off value of a_2, $t(a_1)$, such that the action is in $T(\overline{u})$, and (P) is redundant, if and only if $a_2 \leq t(a_1)$. However, it is not necessarily the case that $T(\overline{u})$ is a decreasing set.

4.2 The First-Order Approach

The reduced problem describes how to optimally implement any given action in $T(\overline{u})$. However, it cannot deal with actions outside $T(\overline{u})$.[14] This problem is addressed here, where a slightly different solution method is developed. This method relies more critically on the assumption that $c_{12} \geq 0$ and in addition necessitates that the principal's benefit function is non-increasing in a_2.

[14] Nevertheless, the reduced problem provides a lower bound on implementations costs outside $T(\overline{u})$. This may be sufficient to rule out that the second-best action is outside $T(\overline{u})$.

Assumption P2 (The benefit function): The principal never benefits from higher a_2, i.e. $B_2(a_1, a_2) \leq 0$ for all (a_1, a_2).

With Assumption P2 in hand, the so-called first-order approach can be justified. The first-order approach minimizes implementation costs subject only to (P) and L-IC, as was discussed before deriving (7). The proof of the validity of the first-order approach utilizes and extends arguments in Rogerson (1985).[15]

Theorem 2 *Assume that A1–A5 and P1–P2 hold and that any second-best action (a_1, a_2) is interior. Then, the first-order approach is valid. The optimal contract is regular and takes the form in (7), with $\mu_1 > 0 > \mu_2$.*

Proof. See the Appendix. ■

5 Conclusion

This paper extends the canonical principal-agent model to allow the agent to pursue private, stochastic, and possibly non-monetary rewards. Conceptually, this way of "unpacking" leisure recognizes that rewards earned while not on the job are also endogenous. Hence, the principal manipulates not only the agent's effort on the job but also his "work-life balance" through the contract design.

The non-separability between rewards from labor income and other sources are at the root of the paper's economic insights. For instance, higher base utility at work reduces the incentive to pursue outside rewards. It is almost as if the agent is more intrinsically motivated as he can be induced to work hard on the job with flatter extrinsic incentives. The non-separability also explains why the agent may earn rent above his reservation utility and why implementations costs may be non-monotonic in effort on the job.

Likewise, at the technical level, it is also this non-separability that represents the main challenge. Thus, the paper's technical contribution is to propose a solution method to deal with private rewards without assuming separability. Recall that separability is implicit in the LEN model, for instance. Thus, the paper's results demonstrate that non-separability have important economic implications.

[15]For more details and discussion see the working paper version, Kirkegaard (2016), where the main focus is on justifying the first-order approach.

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Appendix

Proof of Lemma 1. Integration by parts with respect to x_2 yields

$$EU(a_1,a_2)=\int\left(v(w(x_1),\overline{x}_2)-\int v_2(w(x_1),x_2)G^2(x_2|a_2)dx_2\right)g^1(x_1|a_1)dx_1-c(a_1,a_2). \tag{16}$$

It is well known that Assumption A2 (MLRP) implies that $G^i_{a_i}(x_i|a_i)<0$ for all interior x_i. Moreover, Assumptions A2 and A3 together imply that $G^i_{a_ia_i}(x_i|a_i)>0$ for all $x_i\in(\underline{x}_i,\overline{x}_i)$, $i=1,2$. To see this, note that A3 (LOCC) necessitates that $G^i_{a_ia_i}\geq 0$ and $G^1G^2G^1_{a_1a_1}G^2_{a_2a_2}-\left(G^1_{a_1}G^2_{a_2}\right)^2\geq 0$. At any interior (x_1,x_2), the last term is strictly positive, by A2 (MLRP). Thus, $G^1_{a_1a_1}>0$ and $G^2_{a_2a_2}>0$ are necessary. Since $v_2>0$, the first term in (16) is therefore strictly concave in a_2. The second term is weakly concave in a_2 by Assumption A4. Thus, $EU_{22}<0$.

Assuming the contract is regular, another round of integrating by parts, this time with respect to x_1, yields

$$\begin{aligned}EU(a_1,a_2|w(\cdot)) \;=\;& -\int v_1(w(x_1),\overline{x}_2)w'(x_1)G^1(x_1|a_1)dx_1-\int v_2(w(\overline{x}_1),x_2)G^2(x_2|a_2)dx_2\\ &+\int\int v_{12}(w(x_1),x_2)w'(x_1)G^1(x_1|a_1)G^2(x_2|a_2)dx_1dx_2\\ &+v(w(\overline{x}_1),\overline{x}_2)-c(a_1,a_2).\end{aligned} \tag{17}$$

Thus,

$$EU_{12}(a_1,a_2|w(\cdot))=\int\int v_{12}(w(x_1),x_2)w'(x_1)G^1_{a_1}(x_1|a_1)G^2_{a_2}(x_2|a_2)dx_1dx_2-c_{12}(a_1,a_2).$$

Since $G^i_{a_i}(x_i|a_i)<0$ for all $x_i\in(\underline{x}_i,\overline{x}_i)$, $i=1,2$, the last two parts of Assumption A4, $v_{12}<0$ and $c_{12}\geq 0$, imply that $EU_{12}(a_1,a_2|w(\cdot))<0$ for all regular contracts. A similar argument proves that $EU_{11}(a_1,a_2|w(\cdot))<0$ if the contract is regular.

Since $v_1,v_2>0>v_{12}$ and $G^1(x_1|a_1)$, $G^2(x_2|a_2)$, $G^1(x_1|a_1)G^2(x_2|a_2)$, and $c(a_1,a_2)$ are all convex in $(a_{1,}a_2)$, it follows from (17) that $EU(a_1,a_2|w(\cdot))$ is concave because it is the sum of concave functions. To prove that $EU_{11}EU_{22}-EU_{12}^2>0$ when $w(x_1)$ is regular, let $P(a_1,a_2)$ denote the first line in (17) and let $Q(a_1,a_2)$ denote the remainder, such that $EU=P+Q$. Note that $P_{11},P_{22}<0$

but $P_{12} = 0$. Similarly, $Q_{11}, Q_{22} < 0$ and by concavity $Q_{11}Q_{12} - Q_{12}^2 \geq 0$. Now,

$$\begin{aligned} EU_{11}EU_{22} - EU_{12}^2 &= \left[P_{11}P_{22} - P_{12}^2\right] + \left[Q_{11}Q_{22} - Q_{12}^2\right] + \left[P_{11}Q_{22} + P_{22}Q_{11} - 2P_{12}Q_{12}\right] \\ &= \left[P_{11}P_{22}\right] + \left[Q_{11}Q_{22} - Q_{12}^2\right] + \left[P_{11}Q_{22} + P_{22}Q_{11}\right] > 0, \end{aligned}$$

since the first and third terms are strictly positive and the second term is non-negative. This implies strict concavity. ■

Proof of Proposition 1. For brevity, write EU for $EU(a_1, a_2|w(\cdot))$. Since $w(x_1)$ is regular, Lemma 1 implies that $EU_{11} < 0$, $EU_{22} < 0$, and $EU_{12} < 0$. Moreover, $EU_{11}EU_{22} - EU_{12}^2 > 0$, or

$$\frac{-EU_{12}}{EU_{22}} > \frac{-EU_{11}}{EU_{12}}. \tag{18}$$

Given the agent's problem is concave, the optimal action is determined by the first order conditions $EU_1 = 0$ and $EU_2 = 0$. In (a_1, a_2) space, the curves along which $EU_1 = 0$ and $EU_2 = 0$ have slope

$$\frac{da_2}{da_1}_{|EU_1=0} = \frac{-EU_{11}}{EU_{12}} < 0 \text{ and } \frac{da_2}{da_1}_{|EU_2=0} = \frac{-EU_{12}}{EU_{22}} < 0.$$

The optimal interior action (a_1^*, a_2^*) is found where these two curves intersect. By (18) the curve where $EU_1 = 0$ crosses the curve where $EU_2 = 0$ exactly once, from above. See Figure 3.

Similarly, write EU^ε for $EU(a_1, a_2|\widehat{w}_\varepsilon(\cdot))$. Note that $\widehat{w}_\varepsilon(x_1)$ is also a regular contract. Next, note that by design of $\widehat{w}_\varepsilon(x_1)$, $EU_1^\varepsilon = 0$ at (a_1^*, a_2^*). That is, both the $EU_1 = 0$ curve and the $EU_1^\varepsilon = 0$ curve go through the point (a_1^*, a_2^*). However, by (16) in Lemma 1,

$$\begin{aligned} EU_2(a_1, a_2|w(\cdot)) &= -\int \left(\int v_2(w(x_1), x_2) G_{a_2}^2(x_2|a_2) dx_2 \right) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) \\ &< -\int \left(\int v_2(\widehat{w}_\varepsilon(x_1), x_2) G_{a_2}^2(x_2|a_2) dx_2 \right) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) \\ &= EU_2(a_1, a_2|\widehat{w}(\cdot)), \end{aligned}$$

or $EU_2 < EU_2^\varepsilon$ for all (a_1, a_2). The inequality follows from $v_{12} < 0$, $\widehat{w}_\varepsilon(x_1) <$

$w(x_1)$, and $G^2_{a_2} < 0$. Then, $EU^{\varepsilon}_{22} < 0$ implies that the curve where $EU^{\varepsilon}_2 = 0$ lies above the curve where $EU_2 = 0$. This rules out that $a_1^{'} = a_1^*$. See Figure 3.

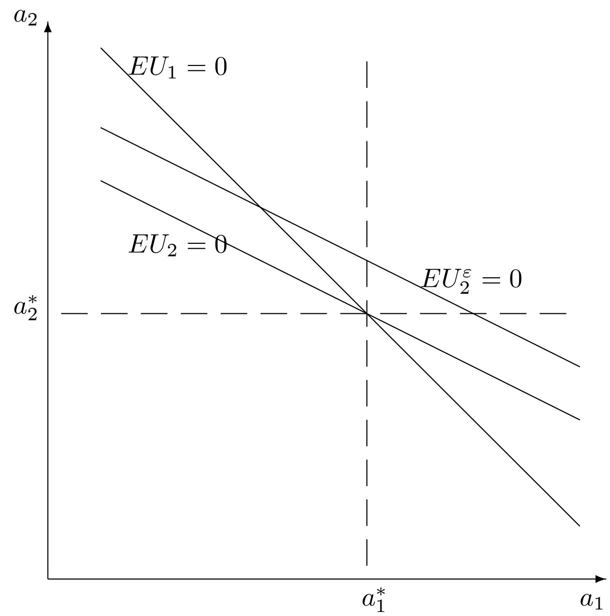

Figure 3: Solving the agent's problem.

Imagine that $a_1^{'} > a_1^*$. Then, since $EU^{\varepsilon}_1 = 0$ crosses the curve where $EU^{\varepsilon}_2 = 0$ from above at (a_1', a_2'), the $EU^{\varepsilon}_1 = 0$ curve must lie above the $EU^{\varepsilon}_2 = 0$ curve for any $a_1 < a_1'$. Thus, $EU^{\varepsilon}_1(a_1^*, a_2^*) > 0$, which is a contradiction. Thus, $a_1^{'} < a_1^*$. As the $EU^{\varepsilon}_2 = 0$ curve lies above the $EU_2 = 0$ curve, it then follows that $a_2' > a_2^*$. ■

Proof of Lemma 2. First,

$$V_1(w, a_2) = \int v_1(w, x_2) g^2(x_2|a_2) dx_2.$$

Assumption A4 (substitutes) implies that $V(w, a_2)$ is strictly increasing and strictly concave in w, or $V_1(w, a_2) > 0 > V_{11}(w, a_2)$, just like a standard utility function. Moreover, A2 and A4 together imply that

$$V_{12}(w, a_2) = \int v_1(w, x_2) g^2_{a_2}(x_2|a_2) dx_2 < 0.$$

The reason is that $v_1(w, x_2)$ is strictly decreasing in x_2 (A4) and that an increase in a_2 leads $G^2(x_2|a_2)$ to become stochastically stronger in the sense of first order stochastic dominance (A2). Similarly, Assumption A4 together with A2 and A3

(LOCC) imply that $V_2(w, a_2) > 0$ and $V_{22}(w, a_2) < 0$, respectively. This proves the first two parts of the lemma.

For the third part, note that A2 (MLRP) is equivalent to the requirement that $g^i(x_i|a_i)$ is log-supermodular in (x_i, a_i), $i = 1, 2$. Then, the term under the integration sign in $V_1(w, a_2)$ is, by A2 (MLRP) and A5 (Log-supermodularity), log-supermodular in *all* (three) arguments, (w, x_2, a_2). In this case, log-supermodularity is preserved under integration so $V_1(w, a_2)$ is log-supermodular in its (two) arguments (w, a_2); see e.g. Athey (2002, p. 193). This proves the third part of the lemma. The fourth part follows from carrying out the differentiation in (12) and using the sign restrictions implied by parts 1 and 2. ■

Proof of Lemma 3. Given $\mu_2 \leq 0$, $V_{11} < 0$ and (12) imply that the right hand side of (7) is strictly increasing in w (the derivative is strictly positive). Thus, for each x_1 there is at most one solution to (7), $w(x_1)$. Differentiability now follows from the differentiability of all the components in (7) and the fact that the right hand side is strictly increasing in w. If $\mu_1 > 0$, Assumption A2 (MLRP) implies that the left hand side is strictly increasing in x_1. Hence, the contract is regular. ■

Proof of Theorem 1. The reduced problem produces a contract of the form in (7), but with $\lambda = 0$. Thus, there exists a value of x_1 for which the left hand side is zero. Any solution to (7) then requires $\mu_2 < 0$. Hence, the right hand side is strictly increasing in w, by A4 and A5. If $\mu_1 \leq 0$, the contract is weakly decreasing, by A2. In this case, $EU_1 < 0$ which violates one of the constraints of the reduced problem. Hence, $\mu_1 > 0$. In summary, the contract is regular by Lemma 3, since $\lambda = 0$ and $\mu_1 > 0 > \mu_2$. By Lemma 1, the contract is incentive compatible. By the assumption imposed on $T(\overline{u})$, (P) is satisfied. Hence, the contract is feasible. ■

Proof of Proposition 3. The objective function in the reduced problem is $-E[w|a_1, a_2]$. By the Envelope Theorem,

$$\frac{\partial E[w|a_1, a_2]}{\partial a_1} = \int w(x_1) g^1_{a_1}(x_1|a_1) dx_1 - \mu_1 EU_{11} - \mu_2 EU_{12} \tag{19}$$

$$\frac{\partial E[w|a_1, a_2]}{\partial a_2} = -\mu_1 EU_{12} - \mu_2 EU_{22} \tag{20}$$

for actions in $T(\overline{u})$. Given (a_1^*, a_2^*), $\mu_1 > 0$ and $\mu_2 < 0$ depend, through L-IC, only on $c_1(a_1^*, a_2^*)$ and $c_2(a_1^*, a_2^*)$ but not on $c_{11}(a_1^*, a_2^*)$, $c_{22}(a_1^*, a_2^*)$, or $c_{12}(a_1^*, a_2^*)$. However, EU_{ij} clearly depends on c_{ij}. Thus, these terms in (19) and (20) can be manipulated by changing (c_{11}, c_{22}, c_{12}). However, Assumption A4 imposes the restrictions that $c_{11}, c_{22}, c_{12} \geq 0$ and $c_{11}c_{22} - c_{12}^2 \geq 0$. In (19), the first two terms are positive while the third term is negative. All three terms are independent of c_{22}. Holding fixed c_{12} and c_{22}, the second term comes to dominate as $c_{11} > 0$ explodes. Thus, there is a cost function for which (19) is strictly positive. On the other hand, fix $c_{11} > 0$ and let $c_{12} > 0$ and $c_{22} > 0$ explode in such a way that the convexity condition $c_{11}c_{22} - c_{12}^2 \geq 0$ remains satisfied. Then, the last term dominates and (19) is strictly negative. This proves the first part of the proposition. Using (20), the second part is proven in a similar manner. ■

Proof of Proposition 4. First, implicit differentiation establishes that $CE_{-V_2}(a_1, a_2)$ is weakly decreasing in a_1 and strictly decreasing in a_2. Second,

$$\begin{aligned}
\frac{\partial \overline{U}(a_1, a_2)}{\partial a_2} &= V_1(CE_{-V_2}, a_2)\frac{\partial CE_{-V_2}}{\partial a_2} + V_2(CE_{-V_2}, a_2) - c_2(a_1, a_2) \\
&= V_1(CE_{-V_2}, a_2)\frac{\partial CE_{-V_2}}{\partial a_2} < 0,
\end{aligned}$$

where the last equality follows from L-IC$_2$. It is readily confirmed that $\overline{U}(a_1, a_2)$ is strictly decreasing in a_1. Thus, if $\overline{U}(a_1, a_2) \geq \overline{u}$ then $\overline{U}(a_1', a_2') \geq \overline{u}$ whenever $(a_1', a_2') \leq (a_1, a_2)$. This proves the second part of the proposition. The last part follows by definition of $T(\overline{u})$. ■

Proof of Proposition 5. For interior actions, the proof is in the text preceding the proposition. For actions involving $a_2 = \overline{a}_2$, incentive compatibility requires $EU_2 \geq 0$. Thus, the contract's certainty equivalent must be at most $CE_{-V_2}(a_1, \overline{a}_2)$. However, if the action is outside $T(\overline{u})$ then such a contract violates the participation constraint since the inequality in (15) becomes an equality in the multiplicative model. ■

Proof of Proposition 6. The only constraint under symmetric information is the participation constraint, which can be made binding. Given the agent is risk averse, the cheapest way to satisfy this constraint is by offering a fixed wage contract. Let $W(a_1, a_2)$ denote the smallest wage that can be offered while enticing the agent to accept a contract that dictates action (a_1, a_2), with

$$V(W(a_1, a_2), a_2) - c(a_1, a_2) = \overline{u}.$$

In comparison, recall that

$$V(CE_{-V_2}(a_1, a_2), a_2) - c(a_1, a_2) \gtrless \overline{u} = V(W(a_1, a_2), a_2) - c(a_1, a_2) \text{ for } a_2 \lessgtr t(a_1)$$

such that

$$CE_{-V_2}(a_1, a_2) \gtrless W(a_1, a_2), a_2) \text{ for } a_2 \lessgtr t(a_1).$$

At the same time,

$$V_2(CE_{-V_2}(a_1, a_2), a_2) - c_2(a_1, a_2) = 0$$

and since $V_{12} < 0$ it now follows that

$$V_2(W(a_1, a_2), a_2) - c_2(a_1, a_2) \gtrless 0 \text{ for } a_2 \lessgtr t(a_1),$$

which in turn implies that

$$\frac{\partial W(a_1, a_2)}{\partial a_2} = -\frac{V_2(W(a_1, a_2), a_2) - c_2(a_1, a_2)}{V_1(W(a_1, a_2), a_2)} \lessgtr 0 \text{ for } a_2 \lessgtr t(a_1).$$

In words, for a fixed a_1, contracting costs are first decreasing in a_2 and then increasing. Thus, costs are minimized at $a_2 = t(a_1)$.

Next, the symmetric-information and partial-information benchmarks differ only in the fact that the latter has an additional constraint, namely that a_2 must be incentivized. Given the extra constraint, contracting costs are at least as high under partial information as under symmetric information. However, $a_2 = t(a_1)$ can be implemented with the exact same fixed-wage contract, where the fixed wage equals $w = W(a_1, t(a_1)) = CE_{-V_2}(a_1, t(a_1))$. Thus, the solution to the partial-information benchmark must coincide with the solution to the symmetric-information benchmark, which in turn coincides with $t(a_1)$. ■

Proof of Proposition 7. The proof follows almost trivially from L-IC$_1$. By definition, $EU_1(a_1, \widehat{a}_2|\widehat{w}(\cdot)) = 0$. Since $EU_{12} < 0$, it then holds that $EU_1(a_1, a_2|\widehat{w}(\cdot)) >$

0 for all $a_2 < \widehat{a}_2$. By similar reasoning, $EU_1(a_1, a_2|\widetilde{w}(\cdot)) < 0$ for all $a_2 > \widetilde{a}_2$. Combining the two yields

$$\begin{aligned}\int V(\widehat{w}(x_1), a_2) l^1_{a_1}(x_1|a_1) g^1(x_1|a_1) dx_1 &\geq c_1(a_1, a_2) \\ &\geq \int V(\widetilde{w}(x_1), a_2) l^1_{a_1}(x_1|a_1) g^1(x_1|a_1) dx_1,\end{aligned}$$

for all $a_2 \in [\widetilde{a}_2, \widehat{a}_2]$, with at least one strict inequality. Since the likelihood-ratio has mean zero, the first term is the covariance between $V(\widehat{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$, while the last term is the covariance between $V(\widetilde{w}(x_1), a_2)$ and $l^1_{a_1}(x_1|a_1)$. This concludes the proof. ■

Proof of Theorem 2. Consider the following *relaxed problem*, so named because the incentive compatibility constraint in the original or "unrelaxed" problem has been weakened,

$$\max_{a_1, a_2, w} B(a_1, a_2) - \int w(x_1) g^1(x_1|a_1) dx_1$$

$$st. \quad EU(a_1, a_2) \geq \overline{u} \tag{P}$$

$$EU_i(a_1, a_2) = 0, \quad i = 1, 2. \tag{L-IC}$$

The first-order approach is said to be valid if the solution to the relaxed problem also solves the original or unrelaxed problem and thus identifies the second-best. As in Rogerson (1985), a *doubly-relaxed* problem is utilized. In Rogerson's one-task model, the relaxed incentive compatibility constraint, $EU_1 = 0$, is replaced with the even weaker constraint that $EU_1 \geq 0$. In the current multi-task model, the appropriate doubly-relaxed problem assumes that

$$EU_1(a_1, a_2) \geq 0 \text{ and } EU_2(a_1, a_2) \leq 0.$$

Rogerson (1985) uses the doubly-relaxed problem to deal with the additional nonlinearities that arise from having a risk averse principal. Here, it is used instead to deal with nonlinearities from the additional incentive constraints.

Conveniently, $\mu_1 \geq 0 \geq \mu_2$ must hold in the doubly-relaxed problem. Moreover, any solution to the doubly-relaxed problem must take the form in (7), with

$\mu_1 \geq 0 \geq \mu_2$. However, wages are constant if $\mu_1 = 0$. Then, $EU_1 = -c_1 < 0$, which violates the doubly-relaxed constraints. Hence, $\mu_1 > 0$ and so $EU_1 = 0$. By Lemma 3, any solution involves a regular contract. By Lemma 1, the agent's problem is concave. The contract is then incentive compatible if $EU_1 = EU_2 = 0$ at the intended action, which holds if $\mu_1 > 0 > \mu_2$.

Thus, the next step establishes that $\mu_2 < 0$. If an interior a_2 is optimal in the doubly-relaxed problem then the principal's first-order condition

$$B_2 + \lambda EU_2 + \mu_1 EU_{12} + \mu_2 EU_{22} = 0, \tag{21}$$

must hold. By Assumption P2, $B_2 \leq 0$. By Lemma 1, it holds that $EU_{12} < 0$ given the contract is regular. Since $\lambda EU_2 \leq 0$, the first three terms in (21) are thus strictly negative. As $EU_{22} < 0$, it is therefore necessary that $\mu_2 < 0$. Hence, $EU_1 = EU_2 = 0$.

By assumption, any second-best action is interior. Thus, any solution to the unrelaxed problem must satisfy $EU_1 = EU_2 = 0$, which implies that it is feasible in the doubly-relaxed problem. However, it has just been shown that any interior solution to the doubly-relaxed problem is feasible in the unrelaxed problem. Hence, the solutions to the unrelaxed and doubly-relaxed problems coincide. Finally, the set of feasible contracts is obviously larger in the doubly-relaxed problem than in the relaxed problem. Then, as the solution to the doubly-relaxed problem involves an interior action, the solution is also feasible in the relaxed problem, which must then identify the exact same solution. This completes the proof.

To clarify, note that the first-order approach justified here does not make it possible to derive implementation costs for actions other than the second-best (contrary to Theorem 1). Thus, it cannot be used as a means to extend the comparative statics in Proposition 3 outside the set $T(\overline{u})$. ■

Online Appendix

The first and longest section of this appendix is devoted to discussing the assumptions of the model. The assumptions are interpreted and in some cases justified more carefully. The relationship to some existing literature is spelled out in more detail and possible relaxations are discussed as well.

The second section discusses boundary actions. The third section presents a reinterpretation of the reduced problem in which the agent is intrinsically motivated to work hard on the job. The fourth and final section examines the model's link to the literature on common agency.

A Assumptions

This section discusses Assumptions A1–A5 in more detail. Examples are given in which A4 and A5 do not hold. One of these examples also clarifies how the model differs from, and is richer than, the Linear-Exponential-Normal model.

A.1 Assumptions A1–A2

Assumption A1 (independence) assumes that the signal x_1 and the private reward x_2 are independent. For example, there is little reason to think that job performance and the mastery of a hobby are correlated. In other settings, such as when the agent is moonlighting in the same industry, the independence assumption is harder to justify. However, the assumption may have some behavioral justification even in such cases. In particular, there is a growing literature on the prevalence and consequences of *correlation neglect*. See e.g. Levy and Razin (2015) and the references therein. In the current context, correlation neglect arises if the principal and the agent know the marginal distributions, but ignore any correlation between the random variables in the joint distribution.

There are at least two technical problems related to relaxing the independence assumption. One is to establish a counterpart to Lemma 1 for "well-behaved" contracts. Moreover, (7) may no longer apply. Thus, it also becomes harder to verify whether the contract is "well-behaved" in the first place. In short, A1 captures the main price of allowing the rewards function to be non-separable.

Assumption A2 (MLRP) ensures both that (i) the contract is regular and that (ii) a first order stochastic dominance property holds, i.e. that $G^i_{a_i}(x_i|a_i) < 0$ for $x_i \in (\underline{x}_i, \overline{x}_i)$. The assumption that $g^1(x_1|a_1)$ is log-supermodular can be replaced with the assumption that the first order stochastic dominance property holds and that there is an exogenous restriction that the contract must be non-decreasing in x_1. Such a restriction arises if the agent can sabotage the signal after it has been realized but before the principal observes it.

The assumption that $g^2(x_2|a_2)$ is log-supermodular plays a role in the aggregation result in Lemma 2. It can be replaced by the first order stochastic dominance property and the more direct assumption that (12) holds. Note that (12) is automatic in the multiplicative model.

A.2 Assumption A3

Assumption A3 (LOCC) is a technically motivated assumption that is instrumental in justifying the solution method. It is a direct extension of Rogerson's (1985) convexity assumption (CDFC). Recall that Rogerson assumes that there is a single signal and a single task. Conlon (2009) presents justifications of the first-order approach (FOA) that permit multiple signals but a single task. Kirkegaard (2017) allows multiple tasks, under the assumption that A1 holds. However, private rewards are ruled out and all signals are contractible. In the previous version of the current paper, Kirkegaard (2016), justifications of the FOA with private rewards are given that are in the spirit of Jewitt's (1988) single-task justifications.

A sufficient condition for LOCC is that G^1 and G^2 are both log-convex. The product of log-convex functions is itself log-convex, and therefore necessarily convex. Alternatively, fix some G^1 that is strictly convex in a_1, but not necessarily log-convex. Then, there is always some "sufficiently convex" G^2 function that ensures that Assumption A3 is satisfied. For example, a non-negative function $h(z)$ is said to be ρ-convex if $h(z)^\rho/\rho$ is convex, or $h''(z)h(z)/h'(z)^2 \geq 1-\rho$ for all z. Thus, a ρ-convex function is log-convex if and only if $\rho \leq 0$ (and convex if and only if $\rho \leq 1$). If $G^2(x_2|a_2)$ satisfies Assumption A2 and is ρ-convex in a_2 (for all x_2) for some small enough ρ (i.e. ρ is negative, but numerically large), then Assumption A3 is satisfied. To see this, note first that the convexity assumption in A3 necessitates that $G^1 G^1_{a_1 a_1}(G^2 G^2_{a_2 a_2}/\left(G^2_{a_2}\right)^2) - \left(G^1_{a_1}\right)^2 \geq 0$ for interior (x_1, x_2). By ρ-convexity, the left hand side is greater than $G^1 G^1_{a_1 a_1}(1-\rho) - \left(G^1_{a_1}\right)^2 \geq 0$. Hence, the inequality is satisfied if ρ is small enough. To reiterate, as long as G^1 satisfies a strict version of CDFC there are G^2 functions that will permit the FOA to be justified even when allowing for private rewards.

There are some similarities between the current model of private rewards and the literature on hidden savings. Ábrahám et al (2011) consider a situation where the agent works for the principal while simultaneously privately investing in a risk-free asset. There is thus no uncertainty concerning the return to the non-contractible action. Hence, performance on the job, x_1, is the only source of uncertainty. Ábrahám et al (2011) justify the FOA by assuming that the distribution of x_1 is log-convex in effort on the job, a_1, and that the agent has decreasing absolute risk aversion. Assumptions A3 (LOCC) and A5 (log-supermodularity) in the current paper can be seen as extensions that allow returns that are both stochastic and potentially non-monetary.

More specifically, let a_2 denote the dollar amount that the agent saves. Savings has a risk-free rate of return of r. Letting $U(\cdot)$ denote the Bernoulli utility function over total wealth, the agent's utility upon earning $w(x_1)$ on the job and ra_2 from savings is $U(w(x_1) + ra_2)$. Given action (a_1, a_2), integration by parts yields expected utility from rewards of

$$\int U(w(x_1)+ra_2)g^1(x_1|a_1)dx_1 = U(w(\overline{x}_1)+ra_2) - \int U'(w(x_1)+ra_2)w'(x_1)G^1(x_1|a_1)dx_1. \tag{22}$$

The first term is concave in (a_1, a_2), given the agent is risk averse. Next, note that decreasing absolute risk aversion in total income is equivalent to log-convexity of $U'(\cdot)$. First, since $U'(\cdot)$ is log-convex in w, the right hand side of the counterpart to (7) is then well-behaved. Second, $U'(\cdot)$ is log-convex in a_2. Then, assuming G^1 is log-convex in a_1, the integrand in the above expression is now the product of functions that are log-convex in (a_1, a_2). Hence, the integrand is log-convex and therefore convex in (a_1, a_2). It now follows that expected utility from rewards are concave in the agent's action. These are the main steps in Ábrahám et al's (2011) justification of the FOA.

Note that log-convexity of $U'(\cdot)$ plays two roles above. Moreover, log-convexity of $U'(\cdot)$ is equivalent to $U'(\cdot)$ being log-supermodular in (w, a_2). In the current paper, $V_1(w, a_2)$ plays the role of $U'(\cdot)$ in (22). Assumption A5 implies that $V_1(w, a_2)$ is log-supermodular in (w, a_2) (Lemma 2). This assumption is used to discipline the FOA contract in (7). However, since $V_1(w, a_2)$ is not necessarily log-convex in a_2, the above argument cannot be used to establish concavity. Instead, concavity in Lemma 1 comes from the convexity assumption in Assumption A3 (LOCC) and the substitutability assumption that $v_{12} < 0$. In A3, convexity also reduces to requiring that the product of two functions, $G^1(x_1|a_1)$ and $G^2(x_2|a_2)$, are convex in (a_1, a_2). Log-convexity of each function is again sufficient.

A.3 Assumptions A4–A5

The assumption in A4 that $v_{12} < 0$ is important in several places, including early on in establishing concavity of the agent's expected payoff (Lemma 1). Relaxing this assumption to allow rewards from different sources to be complements is an important topic for future research but it is likely to be technically challenging.

As discussed in Section 4.1, the reduced problem does not rely on the assumption that $c_{12} \geq 0$. Neither do the following two examples. In fact, the first example does not even require Assumption A1 (independence). This example illustrates why Assumption A4 rules out $v_{12} = 0$. Specifically, the *additive model* has an additively separable rewards function which eliminates any direct interaction between rewards from different sources. As a result, the model is not substantially different from the standard model. The point is that the paper's new results stem from interdependencies in the rewards function. The additive model also effectively reproduces the results of the LEN model.

Example 2 (The Additive Model): Assume that

$$v(w, x_2) = u(w) + q(x_2),$$

where u and q are strictly increasing and strictly concave functions. Note that $v_{12} = 0$. Assume that $c(a_1, a_2)$ is strictly increasing and convex. Recall that a_2 determines the distribution of x_2. Hence, let $Q(a_2)$ denote the expectation of $q(x_2)$, given a_2. By Assumptions A2 and A3, $Q(a_2)$ is strictly increasing and strictly concave. Similarly, a_1 determines the distribution of x_1 and thus the distribution of wages. Let ω denote the contract and write $U(a_1|\omega)$ as the expectation of $u(w(x_1))$, given a_1. Thus,

$$EU(a_1, a_2) = U(a_1|\omega) + Q(a_2) - c(a_1, a_2). \tag{23}$$

Note that for a fixed a_1, the agent's optimal a_2 is unique and independent of the contract. In other words, once the principal has decided which a_1 he wishes to induce, a_2 is predetermined and impossible to manipulate. Henceforth, let $a_2(a_1)$ denote the optimal value of a_2, given a_1. The model is now essentially a standard model since the agent's action is effectively one-dimensional. For concreteness,

$$EU(a_1) = U(a_1|\omega) + Q(a_2(a_1)) - c(a_1, a_2(a_1)).$$

Unsurprisingly, the model has standard features. The principal designs the contract to manipulate a_1. He has to respect the participation constraint that

$$U(a_1|\omega) \geq \overline{u} - Q(a_2(a_1)) + c(a_1, a_2(a_1)).$$

It is easy to verify that the right hand side is increasing in a_1. Thus, the agent must be promised higher rewards from labor income to accept a contract that induces higher effort. To induce interior effort a_1 on the job, L-IC$_1$ is

$$\frac{\partial U(a_1|\omega)}{\partial a_1} = c_1(a_1, a_2(a_1)).$$

Again, it can be checked that the right hand side is increasing in a_1. Thus, to induce higher effort on the job, expected utility from rewards must respond more dramatically to changes in effort. These conclusions are entirely standard.

The LEN model produces identical results. The reason is that the agent's certainty equivalent in the LEN model is separable, as in (23). See Kirkegaard (2016) for a more detailed discussion of private rewards in the LEN model. The chief difference is that the LEN model stipulates that contracts are linear, $w(x_1) = \beta + \alpha x_1$, and that the agent's action is to pick the means of normally distributed signals. Thus, $U(a_1|\omega) = \beta + \alpha a_1$ and $\frac{\partial U(a_1|\omega)}{\partial a_1} = \alpha$. Hence, the LEN model has an extremely convenient one-parameter measure of the strength of incentives, α. The higher α is, the harder the agent works on the job. A drawback of the model in the current paper is that it does not have an equally convenient measure of incentives. On the other hand, the private rewards version of the LEN model is not as flexible since a_2 is predetermined once a_1 has been decided upon. ▲

Assumption A5 is violated in the following model, where utility is quadratic in total income. This gives rise to a rather extreme situation in which (P) is not redundant but the principal has no interest in making it bind.

Example 3 (The Quadratic Model): Assume that the agent's private rewards are monetary and that his utility is quadratic in total income. That is, $v(w, x_2)$ takes the form

$$\begin{aligned} v(w, x_2) &= (w + x_2) - \gamma (w + x_2)^2, \\ &= (w + x_2) - \gamma \left(w^2 + x_2^2\right) - 2\gamma w x_2 \end{aligned}$$

where γ is a strictly positive constant. The domain of $v(w, x_2)$ is a subset of $\mathbb{R}^2_+$ such that the function is strictly increasing and strictly concave in w and in x_2. Clearly, income from the two sources are substitutes. Note, however, that the agent exhibits increasing absolute risk aversion, thus violating Assumption A5.

Let $x(a_2) \geq 0$ denote the expected value of x_2, as a function of a_2. Likewise, let $s(a_2)$ denote the second moment (the expected value of x_2^2) of x_2 as a function of a_2. Both functions are exogenous. By Assumptions A2 and A3, $x(a_2)$ and $s(a_2)$ are strictly increasing and strictly concave. Next, the distribution of x_1 is determined by a_1. Thus, given the endogenous contract, let $\overline{w}(a_1) \geq 0$ denote the expected wage as a function of a_1 and let $\overline{s}(a_1)$ denote the second moment. Then, given x_1 and x_2, the agent's expected utility from action (a_1, a_2) is

$$EU(a_1, a_2) = \overline{w}(a_1) + x(a_2) - \gamma \left(\overline{s}(a_1) + s(a_2)\right) - 2\gamma \overline{w}(a_1) x(a_2) - c(a_1, a_2).$$

The local incentive compatibility constraints are

$$\begin{aligned} \overline{w}'(a_1) - \gamma \overline{s}'(a_1) - 2\gamma \overline{w}'(a_1) x(a_2) &= c_1(a_1, a_2) \\ x'(a_2) - \gamma s'(a_2) - 2\gamma \overline{w}(a_1) x'(a_2) &= c_2(a_1, a_2). \end{aligned}$$

The unique feature of the quadratic model comes from L-IC$_2$. The only endogenous element is the expected income from labor, $\overline{w}(a_1)$. Thus, given the action, *the expected wage is already pinned down from incentive compatibility.* Note that this is all a risk neutral principal cares about. Thus, there is no particular reason to make the participation constraint binding. Hence, for any *fixed* interior action, there is no unique optimal contract and some optimal contracts leave the agent with rent above his reservation utility. Any optimal contract has the same expected wage but differ in the level of risk (as summarized e.g. by the variance, $\overline{s}(a_1) - \overline{w}(a_1)^2$) imposed on the agent. This property is irreconcilable with the standard model. After all, a basic insight from the standard principal-agent model is that risk should not exceed what is required to provide incentives, since the expected wage must normally increase to compensate. That argument, however, relies on a binding participation constraint.

On the other hand, a fixed a_1 can be implemented with a range of different a_2's, each dictating different expected wage costs. To satisfy L-IC$_2$ when a_2 increases, it is necessary that $\overline{w}(a_1)$ decreases. Consequently, if the principal does not care directly about a_2, or $B_2 = 0$, then he will spur the agent to work as hard as possible on accumulating private rewards. Hence, it is in the principal's interest to entice the agent to have a rewarding home life.[16]

Although $\overline{w}(a_1)$ is determined by the incentive constraint on a_2, the agent's marginal costs also depend on a_1. If $c_{12} > 0$, marginal costs with respect to a_2 increases when a_1 increases. To maintain the same level of a_2 the returns to private rewards must be made larger. This is achieved by lowering $\overline{w}(a_1)$. In short, *it is cheaper to induce higher levels of effort on the job*, a_1. Hence, if the agent's effort is productive, or $B_1 > 0$, it is optimal for the principal to induce the agent to work as hard as possible on the job. The reason is, again, that it is

[16] When $\overline{a}_2$ is to be induced, the constraint is that $EU_2 \geq 0$. This is more likely to be satisfied the lower $\overline{w}(x_1)$ is. Thus, the participation constraint is expected to bind.

only the incentive constraint on a_2 that determines contracting costs. ▲

B Boundary actions and three job strata

The paper focuses on interior a_2. If $\overline{a}_2$ is to be induced, incentive compatibility requires that $EU_2(a_1, \overline{a}_2) \geq 0$, or

$$\int \left[-V_2(w(x_1), \overline{a}_2)\right] g^1(x_1|a_1)dx_1 \leq -c_2(a_1, \overline{a}_2).$$

Thus, incentive compatibility only provides an upper bound on CE_{-V_2}. Hence, the principal has more degrees of freedom to meet the participation constraint. For this reason, implementation costs may drop discontinuously at $\overline{a}_2$. In fact, if $(a_1, \overline{a}_2) \in T(\overline{u})$ then any contract that satisfies L-IC$_1$ and makes (P) bind at action $(a_1, \overline{a}_2)$ automatically satisfies $EU_2(a_1, \overline{a}_2) \geq 0$, generally with a strict inequality. Hence, the optimal contract that induces extreme effort in pursuit of private rewards exactly mimics a standard contract, with $\lambda > 0$ but $\mu_2 = 0$.[17]

Theorem 1 assumes that the second-best action is interior. In fact, $a_2 = \overline{a}_2$ cannot be optimal if $B(a_1, a_2)$ decreases sufficiently quickly in a_2 around $\overline{a}_2$, even though implementation costs drop discontinuously at $\overline{a}_2$. This observation creates a stronger tie between Theorems 1 and 2, the latter of which assumes $B_2 \leq 0$. Note also that it may not be possible to implement $a_2 = \overline{a}_2$, by Proposition 5.[18]

The discontinuity is perhaps not surprising given that the standard model also has a discontinuity. In particular, in both the standard and the current model, implementation costs are discontinuous in a_1 at $\underline{a}_1$. Here, however, the discontinuity in a_2 at $\overline{a}_2$ have more interesting economic implications, as follows.

Imagine that $\overline{u}$ is very low, such that $T(\overline{u})$ consists of all actions. Then, implementation costs are not affected by a further decrease in $\overline{u}$ for actions in the interior but they diminish for actions on the $a_2 = \overline{a}_2$ boundary. Thus, when $\overline{u}$ is sufficiently low, the second-best action must feature $a_2 = \overline{a}_2$, such that the

[17] The standard arguments apply. If the agent is already setting $a_2 = \overline{a}_2$ then he cannot increase a_2 further to substitute for lower wages. Thus, the incentives with respect to a_1 are unchanged when $w(x_1)$ is replaced by $\widehat{w}_\varepsilon(x_1)$ as defined in Proposition 1. Hence, (P) binds.

[18] Outside the multiplicative model, even if action $(a_1, \overline{a}_2) \notin T(\overline{u})$ is implementably then it may be prohibitively costly to implement if $c(a_1, \overline{a}_2)$ is very large.

principal can benefit from making the participation constraint bind.

In comparison, assume that $\overline{u}$ is very large, such that $T(\overline{u})$ is empty. In the multiplicative model, Proposition 5 implies that only $a_2 = \underline{a}_2$ can be implemented. In this case, it can be verified that the constraint that $EU_2 \leq 0$ is met when the participation constraint binds. Thus, in the multiplicative model, at least, (P) binds if reservation utility is either very high or very low. Moreover, in these cases the incentive constraint on a_2 is implied by (P). Thus, it is only if $\overline{u}$ is in a middle range that the agent may achieve rent in excess of the reservation utility.

In conclusion, there are three strata of jobs, depending on the agent's reservation utility. Agents with extreme reservation utilities exert either extremely high or extremely low effort in pursuing a rewarding outside life. An agent with a moderate reservation utility is instead manipulated to exert moderate (i.e. interior) effort along this dimension.

C A reinterpretation of the reduced problem

Only L-IC$_1$ and L-IC$_2$ enter the reduced problem. Although these are equality constraint, for the sake of argument imagine weakening L-IC$_2$ by turning it into an inequality constraint, such that the constraints can be written

$$\int V(w(x_1), a_2) g^1_{a_1}(x_1|a_1) dx_1 - c_1(a_1, a_2) = 0 \tag{24}$$

$$\int \left[-V_2(w(x_1), a_2)\right] g^1(x_1|a_1) dx_1 - c(a_1, a_2) \geq -c(a_1, a_2) - c_2(a_1, a_2). \tag{25}$$

In comparison, consider the following contracting problem. First, a_2 is a fixed parameter in the agent's utility function (alternatively, the principal can dictate its value). Second, the agent has a "split personality" when it comes to evaluating the contract. He uses the utility function $V(w, a_2)$ to evaluate incentives but the utility function $-V_2(w, a_2)$ to evaluate the merits of participation. Third, the agent's reservation utility depends on the principal's recommendation, with $\overline{u}(a_1, a_2) = -c(a_1, a_2) - c_2(a_1, a_2)$ describing this reservation utility. Evidently, (24) and (25) define the incentive compatibility constraint and the participation constraint, respectively, in this particular contracting problem.

Note that reservation utility in this model is strictly decreasing in a_1. Thus, the personality that decides on participation is (i) more risk averse and (ii) experiences some kind of self-satisfaction from working hard, thus lowering the threshold for participation when the agent expects to be working hard. Since this side of the agent's personality is more risk averse, it is more put off by any risk included in the contract. On the other hand, it is "intrinsically motivated" to work hard. If this effect is strong enough, implementation costs may be decreasing in a_1, as illustrated in Example 1 and Proposition 3.

Next, note that the standard argument described in Section 2 can be used to prove that the "participation constraint" must bind. Hence, the inequality constraint effectively becomes an equality constraint, as in the reduced problem. Thus, the two models are essentially equivalent for a fixed action.

D Common agency

Given a_2, the principal considers the distribution of private rewards to be fixed. However, the outside rewards are sometimes derived from other principal-agent relationships. This is the case when the agent holds several jobs. In such cases of common agency, principals are strategically interacting with each other.

Bernheim and Whinston (1986) were first to consider such situations. However, they assume that every principal observes the same information. Thus, any principal can observe and verify how well the agent performed for other principals. Bernheim and Whinston (1986) establish that the equilibrium action is implemented at a total cost that coincides with the total cost that would have obtained if the principals could collude (or merge). As Bernheim and Whinston (1986) explain: "We can always view a principal as constructing his incentive scheme in two steps: he first undoes what all the other principals have offered and then makes an 'aggregate' offer [...]. Clearly, if we are at an equilibrium, each principal must, in this second step, select an aggregate offer that implements the equilibrium action at minimum cost." On the other hand, competition between principals typically distorts the equilibrium action away from the second-best.

The model in the current paper instead assumes that outside rewards are private. That is, any given principal cannot observe how well the agent performs for another principal. Holmström and Milgrom (1988) use the term "disjoint observations" to refer to such a setting.

Holmström and Milgrom (1988) use the LEN model to show that the equilibrium action is implemented in a cost-minimizing manner when signals are independent. That is, given independence, Bernheim and Whinston's (1986) result on joint observations extends to disjoint observations in the LEN model. The underlying reason is that independence together with linear contracts and exponential utility imply so much "separability" that nothing is gained from collusion.

Now, the current model does not have the benefit of the same degree of separability. A complete analysis of the common agency problem in this setting is outside the scope of the paper but is planned for future research. However, a natural conjecture is that the equilibrium action is implemented at higher than minimum costs. Thus, the model has a source of distortion that is absent in Bernheim and Whinston (1986) and Holmström and Milgrom (1988).