Contracting with Private Rewards*

René Kirkegaard
Department of Economics and Finance
University of Guelph
rkirkega@uoguelph.ca.

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Abstract

The canonical moral hazard model is extended to allow the agent to face endogenous and non-contractible uncertainty. The agent works for the principal and simultaneously pursues outside rewards. Intrinsic motivation on the job may derive from concerns over "work-life balance," which the principal may manipulate through his contract design. The participation constraint is redundant if reservation utility is low. To an outside observer, the model in this case looks as if the agent uses one personality to evaluate the merits of participation and another to evaluate incentives. Finally, implementation costs may be non-monotonic in effort on the job.

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1 Introduction

The principal-agent model has been tremendously influential in economics. However, the canonical model essentially ignores that a large and often rich part of the agent's life takes place outside the office.

In reality, the agent more often than not faces various forms of endogenous and non-contractible uncertainty outside his place of work. He may pursue a range of potentially rewarding activities that are not directly observable nor necessarily directly relevant to the principal. Moonlighting is a prime example that involves monetary rewards. Examples involving potentially large non-monetary rewards include the agent's health status as impacted by life-style choices, his social status in his peer group, the quality of his match on the marriage market as affected by his search intensity, and so on. The satisfaction from mastering a second language, or any other hobby, is another example.

The aim of this paper is to analyze the consequences of endogenous "private rewards" on optimal contracting. The standard principal-agent model is amended to allow the agent to work on two tasks. The first "task" captures the agent's effort on the job. The second task describes the effort devoted to pursuing private rewards. Only the first task produces a contractible signal.

To help fix ideas, the model is outlined next. The agent's effort on the job, a_1, determines the distribution of a signal, x_1, that the principal observes and contracts upon. The second dimension of the agent's action, a_2, determines the distribution of private rewards, x_2. If the agent takes action (a_1, a_2) and the rewards on and outside the job are w and x_2, respectively, then the agent's Bernoulli utility is

$$v(w, x_2) - c(a_1, a_2), \tag{1}$$

where v is a rewards function and c is a cost function.

A fundamental feature of the model is that the contract impacts not only the agent's choice of a_1 but also his choice of a_2. This is due to the interaction between wages and private rewards in the rewards function. Stated differently, a_2 comes with its own incentive compatibility constraint.

Any given a_1 can typically be implemented with a host of different contracts, each inducing different a_2. Holding a_1 fixed, this creates a link between the contract and the agent's costs. Hence, although (1) is separable in rewards and costs, the underlying incentive compatibility problem means that the agent's costs are an implicit function of the contract, even when there is no change in a_1. An outside observer who believes that a_1 is the agent's only choice variable – as in the standard model – thus misses an important property of the model. Throughout the paper, the biases or misperceptions of such an outside observer are emphasized. A main theme is that by misunderstanding the incentive problem the outsider may interpret the agent's response to contract changes as revealing that intrinsic motivation plays a role in the agent's effort on the job.

Before outlining results, two specific assumptions deserve special mention. First, it is assumed that x_1 and x_2 are independent. This assumption is required for technical reasons, but it also seems a logical starting point for a first paper on private rewards. Independence implies that the results are driven solely by the interactions in the agent's utility function. There are no confounding effects from a motive to manipulate the dependence structure, for example.

Second, some basic structure on the rewards and cost functions are imposed throughout. To motivate, the model should at a minimum capture a setting where private rewards are monetary, resulting from e.g. moonlighting. In this case, rewards from the two different sources are substitutes. Thus, although private rewards are allowed to be non-monetary, the main restriction imposed in the paper is that rewards are always substitutes. Likewise, the two tasks are substitutes in the cost function. Again, this seems natural in the moonlighting case. It should be acknowledged that rewards or actions are sometimes complements. For instance, it is perhaps the case that additional income is better enjoyed if the agent is in good health rather than in bad health. The model does not cover such situations. It is also assumed that the agent is less risk averse with respect to labor income the higher his private reward is. If the private reward is monetary, this assumption reduces to decreasing absolute risk aversion over total income.

In his survey of behavioral contract theory, Kőszegi (2014) singles out "the literature on the interaction between extrinsic and intrinsic motivation [as] one of the most exciting and productive in behavioral contract theory." Intrinsic motivation refers to non-monetary reasons why the agent works hard on the job. Englmaier and Leider (2012) note that if the agent has reciprocal preferences, the principal can "generate intrinsic motivation" by giving the agent higher base utility. The agent reciprocates by maintaining high effort even if explicit incentives are weakened. In the simplest version of Bénabou and Tirole's (2003) model, the agent derives utility if he performs well on the job but he only has an imperfect signal about the cost of effort. If the principal knows that effort is very costly, he may be worried that the agent has received a bad signal. Consequently, he is more likely to offer steeper explicit incentives to partially compensate, yet that may not be enough to prevent the probability of high effort from declining.

In this paper, the contract also does not capture all that is payoff-relevant to the agent. What looks like "intrinsic motivation" to the outsider may reflect that the agent reacts to how rewards on and off the job combine. In this sense, the model endogenizes the agent's pursuit of "work-life balance." The agent simultaneously invests in both "work" and "life" (i.e. activities outside the job). The terms of his employment contract may influence both decisions.

The main economic force is the following. When the contract offers higher employment rewards the agent's incentive to pursue outside rewards diminishes. Then, employment income takes a more dominant role in the agent's overall rewards. Consequently, weaker incentives are required to incentivize him to work hard on the job. In fact, the most profitable way to induce higher effort on the job may entail increasing employment rewards but lessening what the outsider would interpret as extrinsic incentives. The outsider overlooks that the agent's effort in pursuit of outside rewards adjusts downwards at the same time.

The interdependencies between rewards imply that the agent's participation constraint is redundant – it is implied by incentive compatibility – if reservation utility is low enough. Thus, he may earn rent in excess of his reservation utility.[1] In this case the model is isomorphic to a behavioral model where the agent uses two different "personalities" to evaluate the contract. The personality that decides on the merits of participation is more risk averse than the personality that evaluates incentives and decides how hard to work on the job. The agent is then compensated with a higher expected wage than if the latter personality

[1] This is reminiscent of Laffont and Martimort's (2002, Section 5.3) observation that the agent may earn rents when his utility function is non-separable in income and effort. However, the participation constraint is not redundant in their setting. See Section 5.

made both decisions, as in the standard model.

Moreover, the personality that decides on participation derives a measure of self-satisfaction from working hard. Thus, implementation costs may depend on a_1 in an unusual manner. Under common assumptions, implementation costs are strictly increasing in a_1 in the standard model. Intuitively, costlier extrinsic incentives must be provided to induce higher effort. This direct effect carries over to the new model. However, a competing indirect effect is now present as well. In particular, the incentive constraint on a_2 becomes cheaper to satisfy when a_1 increases. After all, it is costlier to engage in outside activities after a long and exhausting day at the office. To reestablish the correct incentives on a_2, the rewards to outside activities must therefore be made to matter more. The principal can achieve this by lowering wages or employment rewards. Equivalently, the personality that decides on participation is happy to work harder and accepts lower compensation. It is as if this personality is intrinsically motivated. An example is presented where the positive indirect effect sometimes dominates the negative direct effect. Thus, implementation costs are locally decreasing in a_1 in the example. Furthermore, if the outside observer misses the indirect effect, then he may conclude that lower effort on the job should be induced.

Highly variable labor income gives the agent the incentive to work harder in pursuit of private rewards. When he works harder at home, he is more likely to earn high private rewards and feel less sensitive to risk in labor rewards. Thus, a flatter wage schedule makes it cheaper to prevent the agent from working too hard outside the office. Hence, private rewards tends to flatten the wage schedule.

Finally, the model is consistent with the casual observation that different strata of jobs with different degrees of work-life balance exists. For instance, agents with very low reservation utility are induced to exert extreme effort towards pursuing outside rewards and thus derive most of their utility from that source. In contrast, agents with very high reservation utilities exert minimal effort outside the job and therefore derives most of their utility from labor income. Agents with moderate levels of reservation utility experience a higher degree of work-life balance. It is this group that may earn rent above reservation utility. An example obtains similar comparative statics when comparing industries that differ in how valuable the agent's marginal effort on the job is to the principal.

2 Model and preliminaries

The agent performs two "tasks", a_1 and a_2, each of which belong to a compact interval, $a_i \in [\underline{a}_i, \overline{a}_i]$, $i = 1, 2$. The first task captures the agent's effort on the job, as a result of which a contractible signal, x_1, is produced. The signal's marginal distribution is $G^1(x_1|a_1)$. The second "task" reflects the agent's pursuit of a private reward. The agent receives a possibly non-monetary reward, x_2, which is determined by the marginal distribution function $G^2(x_2|a_2)$. Assume x_i belongs to a compact interval, $[\underline{x}_i, \overline{x}_i]$, which is independent of a_i. Let $g^1(x_1|a_1)$ and $g^2(x_2|a_2)$ denote the densities and assume that $g^i(x_i|a_i) > 0$ for all $x_i \in [\underline{x}_i, \overline{x}_i]$ and all $a_i \in [\underline{a}_i, \overline{a}_i]$.[2] Note that each marginal distribution depends only on one task. This is further strengthened by assuming that x_1 and x_2 are independent.

Assumption A1 (Independence): *Outcomes are independent*, i.e. given a_1 and a_2, the joint distribution is given by

$$F(x_1, x_2|a_1, a_2) = G^1(x_1|a_1)G^2(x_2|a_2). \tag{2}$$

To continue, define $l^i(x_i|a_i) = \ln g^i(x_i|a_i)$. Let $l^i_{a_i}(x_i|a_i)$ denote the likelihood-ratio, i.e. the derivative of $l^i(x_i|a_i)$ with respect to a_i, $i = 1, 2$, and assume it is bounded. The next assumption is standard in principal-agent models.

Assumption A2 (MLRP): The marginal distributions have the *monotone likelihood ratio property*, i.e. for all $a_i \in [\underline{a}_i, \overline{a}_i]$ it holds that

$$\frac{\partial}{\partial x_i}\left(l^i_{a_i}(x_i|a_i)\right) = \frac{\partial^2 \ln g^i(x_i|a_i)}{\partial a_i \partial x_i} \geq 0 \text{ for all } x_i \in [\underline{x}_i, \overline{x}_i], \tag{3}$$

with strict inequality on a subset of strictly positive measure, $i = 1, 2$.

Assumption A2 implies that $G^i_{a_i}(x_i|a_i) < 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$.[3] The interpretation is that when the agent works harder, bad outcomes are less likely. In particular, if $a_i' > a_i''$ then $G^i(x_i|a_i')$ first order stochastically dominates $G^i(x_i|a_i')$.

[2] Throughout, all exogenous functions are assumed continously differentiable to the requisite degree. For brevity, statements to that effect are omitted from the numbered assumptions.

[3] Recall that $l^i_{a_i}(x_i|a_i)$ is non-decreasing and has expected value of zero. Since $G^i_{a_i}(\underline{x}_i|a_i) = G^i_{a_i}(\overline{x}_i|a_i) = 0$, it follows that $G^i_{a_i}(x_i|a_i) = \int_{\underline{x}_i}^{x_i} l^i_{a_i}(z_i|a_i)g^i(x_i|a_i) < 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$.

Rogerson (1985) justifies the first-order approach (FOA) in a one-signal, one-task model. He assumes that the distribution function is convex in the one-dimensional action. Kirkegaard (2017) allows multiple tasks and signals. He shows that a natural extension is to assume that the distribution function is convex in the many-dimensional action. Although Kirkegaard's (2017) proof does not directly extend, the same assumption is useful here.

Assumption A3 (LOCC): $F(x_1, x_2|a_1, a_2)$ satisfies the *lower orthant convexity condition*; $F(x_1, x_2|a_1, a_2)$ is weakly convex in (a_1, a_2) for all (x_1, x_2) and (a_1, a_2).

Assumption A3 implies that $G^i_{a_i a_i}(x_i|a_i) > 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$, $i = 1, 2$.[4] To progress faster, a more thorough discussion of assumptions are in Section 7.

Assumptions A1–A3 describe the agent's "technology". His preferences are described by Bernoulli utility of the form in (1). The rewards function $v(w, x_2)$ is strictly increasing and strictly concave in each argument, $v_i > 0 > v_{ii}$, $i = 1, 2$, where subscripts denote derivatives.[5] It is assumed that the domain is $\mathbb{R} \times [\underline{x}_2, \overline{x}_2]$, where the assumption that $w \in \mathbb{R}$ ensures that wages are interior.

The paper focuses on the case where rewards and tasks are substitutes. Thus, assume that $v_{12} < 0$; the higher x_2 is, the lower is the marginal utility of additional employment income. Likewise, a_1 and a_2 are weak substitutes in the cost function, or $c_{12} \geq 0$. That is, the marginal cost of increasing a_1 is higher when a_2 is high. The cost function is strictly increasing and jointly convex in (a_1, a_2).

Assumption A4 (Monotonicity/substitutes): The agent's Bernoulli utility is $v(w, x_2) - c(a_1, a_2)$; $v(w, x_2)$ is strictly increasing and strictly concave in each argument separately, with domain $\mathbb{R} \times [\underline{x}_2, \overline{x}_2]$, while $c(a_1, a_2)$ is strictly increasing and weakly convex in (a_1, a_2). Rewards are strict substitutes; $v_{12}(w, x_2) < 0$. Tasks are weak substitutes; $c_{12}(a_1, a_2) \geq 0$.

Given a contract $w(x_1)$ and action (a_1, a_2), the agent's expected payoff is

$$EU(a_1, a_2) = \int\int v(w(x_1), x_2) g^1(x_1|a_1) g^2(x_2|a_2) dx_1 dx_2 - c(a_1, a_2). \quad (4)$$

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[4]LOCC necessitates that $G^i_{a_i a_i} \geq 0$ and $G^1 G^2 G^1_{a_1 a_1} G^2_{a_2 a_2} - \left(G^1_{a_1} G^2_{a_2}\right)^2 \geq 0$. At any interior (x_1, x_2), the last term is strictly positive, by A2. Thus, $G^1_{a_1 a_1} > 0$ and $G^2_{a_2 a_2} > 0$ are necessary.

[5]However, it is not necessary for $v(w, x_2)$ to be jointly concave in (w, x_2). The reason is the same as in Kirkegaard (2017), where it is carefully explained.

For notational simplicity, $EU(a_1, a_2)$ suppresses the dependency on the contract. At this stage, the properties of the endogenous contract are unknown. To get a feel for the problem, however, it is useful to begin by considering the agent's problem if the contract $w(x_1)$ is differentiable and increasing.

Definition (regular contracts): The contract is said to be *regular* if it is differentiable, with $w'(x_1) \geq 0$ for all $x_1 \in [\underline{x}_1, \overline{x}_1]$ and $w'(x_1) > 0$ on a subset of $[\underline{x}_1, \overline{x}_1]$ of positive measure.

Importantly, $EU(a_1, a_2)$ is "well-behaved" when the contract is regular.

Lemma 1 *Assume that Assumptions A1–A4 hold. Then, the agent's expected utility, EU, is strictly concave in a_2, $EU_{22} < 0$. Moreover, if the contract is regular, then EU is strictly concave in (a_1, a_2), with $EU_{11} < 0$ and $EU_{11}EU_{22} - EU_{12}^2 > 0$, and the two tasks are strict substitutes, or $EU_{12} < 0$.*

Proof. See the Appendix. ■

Here, $c_{12} \geq 0$ is used only to prove that the two tasks are substitutes. Note that the agent has a unique optimal action when offered a regular contract.

The principal is risk neutral. Let $B(a_1, a_2)$ denote the direct benefit of the agent's action and assume that it is continuously differentiable. For instance, $B(a_1, a_2)$ could be the expected value of x_1, given a_1. Finally, let $E[w|a_1, a_2]$ denote the expected wage costs if the agent is induced to take action (a_1, a_2).

Assumption P1 (The principal's preferences): The principal is risk neutral, with expected utility $B(a_1, a_2) - E[w|a_1, a_2]$.[6]

It is sometimes necessary to assume that $B(a_1, a_2)$ is non-increasing in a_2. Consider a salesman who represents several companies and who invests effort a_i into understanding firm i's product line. The quantity of firm i's products that he sells depends positively on a_i but negatively on a_j, i.e. on how well he presents competing products.[7] A similar story might hold for real estate agents.

[6] The prefix A and P refer to assumptions that relate to the agent and principal, respectively.

[7] Although firm i's signal (the volume of sales) depends on a_j, the model does accommodate some settings like this. For instance, let $t_i(a_1, a_2) = 2a_i - a_j$ be a parameter in the distribution of x_i, with $a_1, a_2 \in \mathbb{R}$. Thinking of t_1 and t_2 as the choice variables, the cost function is $K(t_1, t_2) = c((2t_1 + t_2)/3, (2t_2 + t_1)/3)$. Then, $K(t_1, t_2)$ is convex if $c(a_1, a_2)$ is convex.

Assumption P2 (The benefit function): The principal never benefits from higher a_2, i.e. $B_2(a_1, a_2) \leq 0$ for all (a_1, a_2).

The principal's problem is to maximize $B(a_1, a_2)$ less wage costs, subject to a participation constraint (P) and incentive compatibility (IC), or

$$\max_{a_1,a_2,w} B(a_1, a_2) - \int w(x_1) g^1(x_1|a_1) dx_1$$

$$st. \quad EU(a_1, a_2) \geq \overline{u} \tag{P}$$

$$(a_1, a_2) \in \arg \max_{(a_1',a_2') \in [\underline{a}_1, \overline{a}_1] \times [\underline{a}_2, \overline{a}_2]} EU(a_1', a_2'). \tag{IC}$$

Here, $\overline{u}$ describes the agent's reservation utility. Any action that solves the problem is referred to as a second-best action. If a second-best action is interior, incentive compatibility necessitates that expected utility achieves a stationary point at that action, or $EU_1 = 0 = EU_2$. These constraints are referred to as the "local" incentive compatibility constraints. The shorthand L-IC$_i$ will be used to refer to the constraint that $EU_i = 0$, $i = 1, 2$, while L-IC refers to the constraint that L-IC$_1$ and L-IC$_2$ are satisfied at the same time. The first parts of the paper focus on interior second-best actions. Boundary actions are examined later.

Consider the following *relaxed problem*, so named because the incentive compatibility constraint in the original or "unrelaxed" problem has been weakened,

$$\max_{a_1,a_2,w} B(a_1, a_2) - \int w(x_1) g^1(x_1|a_1) dx_1$$

$$st. \quad EU(a_1, a_2) \geq \overline{u} \tag{P}$$

$$EU_i(a_1, a_2) = 0, \quad i = 1, 2. \tag{L-IC}$$

The FOA is said to be valid if the solution to the relaxed problem also solves the original or unrelaxed problem. Lemma 1 implies that the agent's expected utility is concave if the solution to the relaxed problem entails a regular contract. Then, L-IC is not only necessary but also sufficient. In this case, the FOA is valid.

Unfortunately, it is not trivial to establish that $w(x_1)$ is regular. This is in fact the paper's primary technical challenge. However, the overriding objective is to understand the implications of contracting with private rewards. To this end, Section 3 begins by comparing the model with simpler benchmarks.

3 Benchmarks and constraints

To bring out the unique features of the model, it is useful to contrast it to simpler benchmarks. Some preliminary observations on intrinsic motivation follow.

3.1 Benchmarks and the participation constraint

One of the fundamental lessons from the "standard model" with a single task and separable utility is that the agent's participation constraint is binding. A common argument proceeds as follows. Let a be the agent's action, x the signal, $u(w)$ the reward function, and $k(a)$ the cost function. Consider a contract $w(x)$ that gives the agent rent above his reservation utility. Let $E[u(w(x))|a]$ denote the agent's expected utility from rewards, given action a. Now construct another contract, $\widehat{w}_\varepsilon(x)$, defined such that $u(\widehat{w}_\varepsilon(x)) = u(w(x)) - \varepsilon$, where $\varepsilon > 0$. Given ε is small enough, the agent still accepts the contract. Moreover, the incentives are unchanged because $E[u(\widehat{w}_\varepsilon(x))|a] - k(a)$ is maximized where $E[u(w(x))|a] - k(a)$ is maximized. Note that this is due to separability between the contract and the cost function. Thus, the agent delivers the same effort with either contract. However, the principal is better off since $\widehat{w}_\varepsilon(x) < w(x)$ for all x. Thus, any contract that does not make the participation constraint bind cannot be optimal.

An identical argument holds in the outside rewards setting with multi-tasking if x_2 is contractible. Starting from an original contract $w(x_1, x_2)$, a new contract can be constructed where $v(\widehat{w}_\varepsilon(x_1, x_2), x_2) = v(w(x_1, x_2), x_2) - \varepsilon$ for all (x_1, x_2). Thus, a contract cannot be optimal unless the participation constraint binds. The fact that x_2 also contributes directly to the agent's well being is irrelevant to the argument. Intuitively, when the outside reward is contractible, the principal can appropriate the monetary value of the outside reward and then use the contract as the only source of incentives. The standard argument then applies.

However, this simple argument does not extend to contracting with outside rewards when those rewards are private and non-contractible. One clue comes from the observation in the introduction that private rewards implicitly destroy separability between the contract and the cost function. A related observation is that it is generally impossible to construct a counterpart to $\widehat{w}_\varepsilon$ in the new model. That is, there is generally no $\widehat{w}_\varepsilon(x_1)$ function for which $v(\widehat{w}_\varepsilon(x_1), x_2) =$

$v(w(x_1), x_2) - \varepsilon$ for all (x_1, x_2). Thus, the argument does readily extend.

Nevertheless, there is an exception. If $v(w, x_2)$ is additively separable, with $v_{12} = 0$, then $\widehat{w}_\varepsilon(x_1)$ is easily constructed. Thus, the technical challenge – and a main contribution of the paper – is to appropriately deal with non-separability or interactions in the rewards function. Section 7 documents that the popular Linear-Exponential-Normal (LEN) model essentially reduces to a separable model. Recall that Assumption A4 imposes $v_{12} < 0$.

For the next benchmark, assume that x_2 is not contractible but that a_2 is exogenously fixed. This is a minute extension of the standard model since the agent is now once again working on a single task. In particular, define

$$V(w, a_2) = \int v(w, x_2) g^2(x_2|a_2) dx_2 \tag{5}$$

as the expected utility of a fixed w, given that the agent exerts effort a_2 towards obtaining private rewards. Note that the expectation is over x_2, given a_2. Then,

$$EU(a_1, a_2) = \int V(w(x_1), a_2) g^1(x_1|a_1) dx_1 - c(a_1, a_2). \tag{6}$$

Thus, when a_2 is exogenous, the model reduces to a standard one-task model with V in place of v. The participation constraint is binding. This follows from the same type of argument as given above because it is possible to construct a contract for which $V(\widehat{w}_\varepsilon(x_1), a_2) = V(w(x_1), a_2) - \varepsilon$. Moreover, given Assumptions A1–A3, standard arguments prove that the FOA is valid, see Rogerson (1985, footnote 8). For future reference, note also that implementation costs in this case are strictly increasing in a_1, see e.g. Jewitt et al (2008). The "outside observer" is assumed to have a model of this nature in mind. A major theme of the paper is to identify the mistakes in inference made by such an outsider. The outsider is often assumed to be able to observe a_1 and perhaps also a_2, even though the principal cannot verify either in a court of law.

Returning to the main model, where a_2 is endogenous, the contract $\widehat{w}_\varepsilon(x_1)$ constructed from $w(x_1)$ in the previous paragraph directly impacts the agent's choice of a_2. In particular, since $\widehat{w}_\varepsilon(x_1) < w(x_1)$ and wages and private rewards are substitutes, the agent is now incentivized to work harder in pursuit of private rewards. The change in a_2 then generally causes the agent's optimal level of a_1 to change as well. In short, the new contract fails to keep the agent's incentives unchanged. As before, the standard argument used to prove that (P) binds does not extend. In fact, it is shown in Section 5 that (P) may be redundant.

3.2 Incentive compatibility and intrinsic motivation

A first illustration of why the model is relevant for the literature on intrinsic and extrinsic motivation can now be provided. Consider the following thought experiment. An outside observer believes a_2 is endogenously fixed. He observes the contract, $w(x_1)$, as well as the action, (a_1, a_2). The outsider has enough information to compute $V(w(x_1), a_2)$ but he does not know $B(a_1, a_2)$, $\overline{u}$, or $G^1(x_1|a_1)$. Now imagine the outside option deteriorates. Although the outsider is unable to derive the new optimal contract, he can derive $\widehat{w}_\varepsilon(x_1)$ where $V(\widehat{w}_\varepsilon(x_1), a_2) = V(w(x_1), a_2) - \varepsilon$ as before. He knows that the agent will still sign the contract for small enough ε, since $\overline{u}$ has decreased. He also believes that incentives are unchanged, although the new contract pays less. Thus, he would be ready to recommend $\widehat{w}_\varepsilon(x_1)$ as being an improvement over the old contract.

Assume for concreteness that contracts are regular. This assumption is justified in Sections 4 and 5. As mentioned previously, the agent responds to the contract change by increasing a_2. Since the two tasks are strict substitutes when the contract is regular (Lemma 1), the increase in a_2 then causes a_1 to decrease. This is proven formally in the next proposition.

Proposition 1 *Assume that A1–A4 hold. Fix a regular contract, $w(x_1)$, and let (a_1^*, a_2^*) denote the interior action that it induces. If the agent participates, let (a_1', a_2') denote the action that is induced when $w(x_1)$ is replaced by $\widehat{w}_\varepsilon(x_1)$ where $V(\widehat{w}_\varepsilon(x_1), a_2) = V(w(x_1), a_2) - \varepsilon$, $\varepsilon > 0$. Then, $a_1' < a_1^*$ and $a_2' > a_2^*$.*

Proof. See the Appendix. ■

Recall that the outsider believes that $\widehat{w}_\varepsilon(x_1)$ provides the same incentives as $w(x_1)$. Thus, he is surprised when the agent is observed to lower his effort at work, especially if the outsider is not paying attention to the fact that a_2 has changed at the same time (he has little reason to pay attention to a_2 since he thinks it it fixed). The point is that the outsider may attribute this change in behavior to a deterioration in intrinsic motivation and think that the agent is punishing the principal for offering a contract that pays lower wages.

Continuing this discussion, compare the regular contract $w(x_1)$ from Proposition 1 with some other regular contract, $\widetilde{w}(x_1)$, that always pays less, or $\widetilde{w}(x_1) < w(x_1)$. As before, this tends to increase a_2 which then creates downwards pressure on a_1. The aim of the next result is to formalize the idea that in order to induce the same effort on the job, $\widetilde{w}(x_1)$ needs to have stronger incentives than $w(x_1)$. For this purpose, a measure of "stronger incentives" is required. Incentives are derived from the fact that the agent can manipulate expected utility by changing his effort on the job. This suggests that the size of $EU_1(a_1^*, a_2^*)$ measures the power of incentives.[8] With this in mind, the next result shows that $EU_1(a_1^*, a_2^*) > 0$ is necessary for the agent's effort on the job to remain unchanged at a_1^* when pay is lowered. The outsider would expect $EU_1(a_1^*, a_2^*) > 0$ to imply that higher effort on the job is exerted; the fallacy is that a_2 does not remain fixed at a_2^*. Thus, the result can be interpreted as saying that when pay is lowered, $\widetilde{w}(x_1) < w(x_1)$, the agent's intrinsic motivation weakens and extrinsic motivation must be strengthened to compensate.

Proposition 2 *Assume that A1–A4 hold. Fix a regular contract, $w(x_1)$, and let (a_1^*, a_2^*) denote the interior action that it induces. If the agent participates, let (a_1', a_2') denote the action that is induced when $w(x_1)$ is replaced by another regular contract $\widetilde{w}(x_1)$, where $\widetilde{w}(x_1) < w(x_1)$. If $a_1' = a_1^*$ then $EU_1(a_1^*, a_2^*) > 0$ under the contract $\widetilde{w}(x_1)$.*

Proof. See the Appendix. ■

Proposition 2 perhaps suggests that it can never be optimal for the participation constraint to be slack, at least if the principal does not directly care about a_2. It seems cheaper to induce higher a_2 while keeping a_1 unchanged. However, the caveat is that there may exist no contract like $\widetilde{w}(x_1)$ that successfully restores

[8] Since contracts are typically non-linear in this setting, the slope $w'(x_1)$ is not unique and thus cannot be used to succinctly describe incentives; see Section 7. Nevertheless, Section 4.3 does discuss the slope of the contract in more detail.

the incentive to maintain a_1 when pay is lowered.[9] When this is the case, any two contracts that induce the same a_1 must cross. Then, it may be very costly to provide the steeper incentives that are required to maintain the same a_1 along with a higher a_2. In fact, it will be shown later that implementation costs may be non-monotonic in a_2, holding a_1 fixed.

4 Optimal contracts

This section begins by introducing a crucial additional assumption on the rewards function. The assumption has a natural economic interpretation. At the technical level, it adds structure to any contract derived from the FOA. The multipliers to the incentive constraints are interpreted and their significance discussed. Finally, the FOA is justified and the shape of the optimal contract is examined.

4.1 Aggregation and FOA contracts

It is useful to utilize the aggregate function $V(w, a_2)$. This function inherits many properties from its constituent parts. For instance, it is increasing in both arguments. Nevertheless, slightly more structure is required to proceed.

Assumption A5 (Log-supermodularity): The agent's marginal utility of labor income, $v_1(w, x_2)$, is log-supermodular in (w, x_2), or

$$\frac{\partial^2 \ln v_1(w, x_2)}{\partial w \partial x_2} \geq 0 \text{ for all } w \text{ and all } x_2 \in [\underline{x}_2, \overline{x}_2]. \tag{7}$$

Assumption A5 is equivalent to assuming that

$$\frac{\partial}{\partial x_2}\left(\frac{-v_{11}(w, x_2)}{v_1(w, x_2)}\right) \leq 0 \text{ for all } w \text{ and all } x_2 \in [\underline{x}_2, \overline{x}_2]. \tag{8}$$

Thus, the agent's absolute risk aversion over labor income is decreasing in x_2. In

[9] Jewitt et al (2008) make a similar observation in a standard model amended to include upper and lower bounds on payments. Here, $w(x_1)$ takes the role of an upper bound on $\widetilde{w}(x_1)$. At the same time, L-IC$_2$ implicitly limits how much wages can be lowered. If wages are very low, the agent will work harder at home, increasing a_2 beyond what is intended. This argument does not apply if $\overline{a}_2$ is to be induced, but in this case (P) might pose a problem.

other words, the agent is less sensitive to risk in labor income when the private reward is high. If the private reward is monetary and $v(w, x_2)$ takes the form $u(w + x_2)$, then Assumption A5 is equivalent to the assumption that the agent has decreasing absolute risk aversion with respect to total income.

Note that Assumption A2 (MLRP) is equivalent to the requirement that $g^i(x_i|a_i)$ is log-supermodular in (x_i, a_i), $i = 1, 2$. As described by e.g. Athey (2002), log-supermodularity is preserved under integration. Thus, combining A2 and A5 means that the risk preferences of $v(w, x_2)$ aggregates as well. The following lemma characterizes the properties of $V(w, a_2)$.

Lemma 2 *Given A1–A5, the function $V(w, a_2)$ has the following properties:*

1. *$V(w, a_2)$ is strictly increasing and strictly concave in both arguments,*

$$V_i(w, a_2) > 0 > V_{ii}(w, a_2), \quad i = 1, 2.$$

2. *w and a_2 are strict substitutes, $V_{12}(w, a_2) < 0$.*

3. *$V_1(w, a_2)$ is log-supermodular in (w, a_2),*

$$\frac{\partial}{\partial a_2}\left(\frac{-V_{11}(w, a_2)}{V_1(w, a_2)}\right) = \frac{\partial}{\partial w}\left(\frac{-V_{12}(w, a_2)}{V_1(w, a_2)}\right) \leq 0. \tag{9}$$

4. *Parts 1–3 imply that $V_{112}(w, a_2) > 0$.*

Proof. See the Appendix. ■

Turning to the FOA contract, let $\lambda \geq 0$ denote the multiplier to the participation constraint. Let μ_1 and μ_2 denote the multipliers to L-IC$_1$ and L-IC$_2$, respectively, in the relaxed problem. The optimal wage if x_1 is observed is implicitly characterized by the necessary first order condition

$$\lambda + \mu_1 l^1_{a_1}(x_1|a_1) = \frac{1}{V_1(w, a_2)} - \mu_2 \frac{V_{12}(w, a_2)}{V_1(w, a_2)}. \tag{10}$$

Note that (10) differs from its counterpart in the standard model only by its last term, which is where the new L-IC$_2$ constraint is taken into account. It is clear that (9) imposes useful structure on this term.

The properties of the contract that solves (10) depend on the signs of the two multipliers μ_1 and μ_2, which at this point are unknown. However, it is noted that the contract $w(x_1)$ is well-behaved when $\mu_1 \geq 0 \geq \mu_2$.

Lemma 3 *Given A1-A5 and P1, $w(x_1)$ as defined in (10) is unique whenever $\mu_1 \geq 0 \geq \mu_2$. Moreover, $w(x_1)$ is regular if $\mu_1 > 0 \geq \mu_2$.*

Proof. See the Appendix. ■

Combining Lemmata 1 and 3 implies that the FOA is valid if $\mu_1 > 0 \geq \mu_2$. Technically, then, the problem of signing these multipliers are central to justifying the FOA and characterizing the optimal contract. However, it is important to recognize that the signs of the multipliers have economic implications and interpretations as well. In fact, Rogerson (1985) argues that when only a_1 is endogenous, $\mu_1 \geq 0$ is intuitive because it seems plausible that the real incentive problem is to prevent the agent from shirking (lowering a_1) rather than working too hard on the job (increasing a_1). Similarly, $\mu_2 \leq 0$ when the problem is to prevent the agent from working harder outside the job than the principal intends.

It is worth being more explicit about the role of the multipliers in determining implementation costs and the second-best action. If the FOA is valid, the second-best action is determined by the principal's first order conditions

$$B_1 - \int w(x_1) g^1_{a_1}(x_1|a_1) dx_1 + \lambda EU_1 + \mu_1 EU_{11} + \mu_2 EU_{12} = 0. \tag{11}$$

and

$$B_2 + \lambda EU_2 + \mu_1 EU_{12} + \mu_2 EU_{22} = 0, \tag{12}$$

when the second-best action is interior. Of course, $\lambda EU_i = 0$, $i = 1, 2$, in the relaxed problem. The first term in (11) and (12) describe the direct benefit to the principal of increasing a_1 and a_2, respectively. The second term in (11) captures the direct increase in costs from an increase in a_1 that comes from that fact that higher signal realizations and thus higher wage payments are more likely to occur. The purpose of the following discussion is to highlight the importance of the last two terms in (11) and (12). These terms describe the *overall change* in costs arising from the need to maintain incentive compatibility when a_1 and a_2

changes. Recall that L-IC has two components, L-IC$_1$ and L-IC$_2$, and that *both* are affected when a_i is changed but a_j is held constant, $i \neq j$.

In (11), the term $\mu_1 EU_{11}$ reflects the direct cost from having to adjust the contract to satisfy L-IC$_1$ when a_1 changes. The term $\mu_2 EU_{12}$ captures an indirect effect: The cost of satisfying L-IC$_2$ changes when a_1 changes. If $\mu_1 > 0 > \mu_2$ then the first term is negative and thus contributes towards lowering the principal's profit. As in the standard model, it is costlier to induce higher effort at work, other things being equal. The second term is positive, benefitting the principal. Stated differently, it becomes *cheaper* to maintain the same incentives outside the job when the agent works harder on the job. Since the two tasks are substitutes, it is just less tempting (marginal rewards are lower and marginal costs are higher) to exert more effort at home after a hard but rewarding day at the office.

In (12), $\mu_2 EU_{22} > 0$ captures the direct effect. Specifically, it is cheaper to induce the agent to work hard at home, other things being equal. Indeed, the principal can entice the agent to care more about outside rewards by *lowering* wages. The term $\mu_1 EU_{12} < 0$ is an indirect effect: It is harder to maintain the agent's interest in his job if he has a very rewarding home life. Stated differently, stronger and more costly incentives are required to maintain L-IC$_1$ when a_2 increases. This mirrors the discussion that followed Proposition 2.

In short, there is a "ripple effect" from changing the level of a_i that is induced, $i = 1, 2$. Specifically, the cost of maintaining the correct incentives on a_j changes as well, $j \neq i$. In either case – whether a_1 or a_2 is manipulated – the direct and indirect effects pull in *opposite directions* in terms of the overall change in implementation costs when $\mu_1 > 0 > \mu_2$. One implication is that implementation costs should not necessarily be expected to be monotonic in a_1 or a_2.

Proposition 3 *Assume A1–A5 and P1 hold. If the FOA is valid and $\mu_1 > 0 > \mu_2$ then the direct and indirect effects on implementation costs pull in opposite directions as a_i changes marginally from any interior second-best action, $i = 1, 2$.*

Proof. Combining Lemmata 1 and 3 implies that $\mu_1 EU_{11} < 0 < \mu_2 EU_{12}$ and $\mu_1 EU_{12} < 0 < \mu_2 EU_{22}$ at any interior second-best action. ■

The above direct and indirect effects are intuitive, inspiring confidence that $\mu_1 > 0 > \mu_2$ may in fact hold and thus in turn justify the FOA.

4.2 Justifying the FOA

The preceding discussion leads to the "guess" that $\mu_1 > 0 > \mu_2$. Inspired by Rogerson (1985), the idea here is to verify whether such a guess is correct. Thus, as in Rogerson (1985), a doubly-relaxed problem is utilized.

In Rogerson's one-task model, the relaxed incentive compatibility constraint, $EU_1 = 0$, is replaced with the even weaker constraint that $EU_1 \geq 0$. In the current multi-task model, the appropriate doubly-relaxed problem assumes that

$$EU_1(a_1, a_2) \geq 0 \text{ and } EU_2(a_1, a_2) \leq 0.$$

Rogerson (1985) uses the doubly-relaxed problem to deal with the additional nonlinearities that arise from having a risk averse principal. Here, it is used instead to deal with nonlinearities from the additional incentive constraints.

Conveniently, $\mu_1 \geq 0 \geq \mu_2$ must hold in the doubly-relaxed problem. The proof of the following theorem uses (12) and, critically, invokes Assumption P2 to conclude that the multipliers must be non-zero. This in turn means that the incentive constraints bind. Hence, the solution is feasible in both the relaxed problem and the original problem, both of which it must thus solve. Conversely, any solution to the relaxed problem is also feasible in the doubly-relaxed problem. The conclusion is that the FOA is valid if the second-best action is interior.

Theorem 1 *Assume that A1–A5 and P1–P2 hold and that any second-best action (a_1, a_2) is interior. Then, the FOA is valid, with $\mu_1 > 0 > \mu_2$.*

Proof. See the Appendix. ■

Theorem 1 provides conditions under which the FOA is valid and the optimal contract is regular. Thus, the intuition developed in Propositions 1–3 apply.

4.3 The shape of the optimal contract

The last term in (10) works in the direction of making the optimal contract flatter. The intuition is as follows. First, since $\mu_2 < 0$, the incentive problem is to prevent the agent from pursuing outside rewards too enthusiastically. By (9), the agent dislikes risk in labor income less when a_2 is high. Thus, when the agent faces more risk at work, he is more tempted to work hard at home because then he would care less about the risk he is exposed to on the job. That is, the incentive constraint on a_2 becomes harder to satisfy. The wage schedule is thus made flatter to make it cheaper to satisfy L-IC$_2$.

Thus, it will be argued that the contract in (10) is flatter than the contract in the standard problem where a_2 is exogenous, other things equal. The issue is to make precise what "other things equal" entails in this setting.

One thing that is almost certainly not equal in the two problems is implementation costs. The standard problem has fewer constraints since it ignores L-IC$_2$. Hence, given the same action, wage levels are expected to be lower in the standard problem. Since marginal utility is therefore higher, a flatter wage schedule can still promise the same marginal increase in utility from higher effort on the job.[10] In other words, the level effect tends to make the contract flatter in the standard problem. Thus, there are competing effects from L-IC$_2$. Likewise, the optimal value of a_1 may differ in the two problems, as may λ and μ_1.

Thus, to hold "other things equal," another thought experiment is proposed. The universe of standard models is searched until one is found that has the same a_1 and μ_1 and the same expected wage costs as in the real problem. More specifically, c_1 is changed in such a way that the multiplier to L-IC$_1$ in the new problem ends up coinciding with μ_1 in the real problem. At the same time, $\overline{u}$ is adjusted until the contract in the new problem, $s(x_1)$, entail the same expected wage costs as the original contract from (10), $w(x_1)$.[11] Finally, $B(a_1, a_2)$ is manipulated to ensure that the second-best action entails the same a_1 in the standard problem.

In summary, the two problems now feature the same action, share the same multiplier on L-IC$_1$, and have the same implementation costs. The rewards function is the same in both problems, but L-IC$_2$ is present in only one of them. Now, for any signal realization where $w(x_1) = s(x_1)$, it must be the case that $w(x_1)$ is strictly flatter than $s(x_1)$.[12] Hence, the two contracts cross exactly once,

[10] For example, $\widehat{w}_\varepsilon(x_1)$ in Section 3 is easily verified to be flatter than $w(x_1)$ when $\varepsilon > 0$.

[11] Intuitively, μ_1 measures the steepness of the incentives or wage schedule in the standard problem. Incentives must be steep enough to justify the marginal costs of effort. Letting c_1 vary from zero to infinity, it should then be possible to find some c_1 that gives rise to the desired level of incentives, or μ_1. Similarly, the wage level responds to $\overline{u}$ in the standard model.

[12] First, $s(x_1)$ and $w(x_1)$ must cross at least once since they have the same expectation. Second, note that the left hand side of (10) changes equally fast in x_1 in both problems, since

with $w(x_1)$ being flatter than $s(x_1)$ at the point of crossing. Thus, "other things equal" the addition of private rewards flattens the wage schedule.

Recall that the effect from the last term in (10) is always present, even when μ_1 is not exactly the same in the two problems. The principal always faces a trade-off between the level of compensation and the level of risk in terms of satisfying L-IC$_2$. Both higher risk and lower wages make it more tempting to work harder outside the job. Thus, by decreasing risk, wages can be safely lowered a bit. This is of course similar to how the participation constraint is cheaper to satisfy when less risk is involved. Hence, there is a deeper link between (P) and L-IC$_2$.

5 A reduced problem

This section delves deeper into the problem by examining the interaction between incentive compatibility and the participation constraint. It turns out that the participation constraint may be redundant. The section culminates in another justification of the FOA and another look at intrinsic motivation.

5.1 Incentive compatibility

Note that L-IC$_2$ can be written as

$$\int \left[-V_2(w(x_1), a_2)\right] g^1(x_1|a_1)dx_1 = -c_2(a_1, a_2).$$

The function $[-V_2(w, a_2)]$ is strictly increasing and strictly concave in w, since $V_{12} < 0$ and $V_{112} > 0$. Thus, it can be thought of as a kind of utility function in its own right. Under this interpretation, L-IC$_2$ then nails down the certainty equivalent, as evaluated by $[-V_2(w, a_2)]$, of any contract that induces an interior a_2. When it exists, let this be denoted $CE_{-V_2}(a_1, a_2)$, with

$$-V_2(CE_{-V_2}(a_1, a_2), a_2) = -c_2(a_1, a_2). \tag{13}$$

μ_1 is the same by construction. For any given wage, the right hand side increases faster in w when $\mu_2 < 0$ than when $\mu_2 = 0$, where $\mu_2 = 0$ describes the standard model. To compensate, $w'(x_1) < s'(x_1)$ when $w(x_1) = s(x_1)$.

Evidently, if $CE_{-V_2}(a_1, a_2)$ does not exist for some interior action, then that action cannot be implemented. Given existence, however, it is important to understand the properties of $CE_{-V_2}(a_1, a_2)$ because it summarizes how the contract must adapt to provide the correct incentives for pursuing private rewards.

Lemma 4 *Assume A1–A5 hold and that* $CE_{-V_2}(a_1, a_2)$ *as defined in (13) exists. Then,* $CE_{-V_2}(a_1, a_2)$ *is strictly decreasing in* a_1 *and* a_2*.*

Proof. The result follows from implicit differentiation (omitted). ■

Lemma 4 relies critically on the fact that tasks are substitutes. To understand why $CE_{-V_2}(a_1, a_2)$ is decreasing in a_2, note that the value of the contract must be lowered to spur the agent to take a greater interest in pursuing private rewards.

The reason that $CE_{-V_2}(a_1, a_2)$ is decreasing in a_1 is more intricate, but no less important. First, recall that $CE_{-V_2}(a_1, a_2)$ is derived from the incentive constraint with respect to a_2. Imagine the principal wishes to induce a higher a_1 without changing a_2. The increase in a_1 increases the marginal cost of a_2, thus making the agent reluctant to maintain the same a_2. To reestablish incentives, the marginal return to a_2 must be increased. This is achieved by lowering compensation at work, as measured by $CE_{-V_2}(a_1, a_2)$. This effect is a manifestation of the ripple effect described in Section 4.

5.2 Contractible effort on the job

To further understand how L-IC$_2$ impacts the problem, assume temporarily that effort on the job, a_1, is contractible. This does not completely eliminate the moral hazard problem, however, as a_2 still needs to be incentivized. In other words, L-IC$_2$ is in play but L-IC$_1$ is not. Recall that Lemma 4 depends only on L-IC$_2$.

A contract now specifies a wage schedule $w(x_1)$ as well as a required level of effort on the job. Given there is no need to incentivize a_1, a fixed-wage contract can be used. To satisfy L-IC$_2$ and incentivize an interior a_2, the fixed wage must be set equal to $CE_{-V_2}(a_1, a_2)$. From Lemma 4, implementation costs are thus strictly *decreasing* not only in a_2 but also in a_1. Hence, if the principal directly benefits from a_1 but not from a_2, then higher actions are unambiguously more profitable for the principal. Of course, the principal's ability to manipulate the action is limited by the participation constraint, which is taken into account next. However, it is worth first emphasizing that even when a_1 is made non-contractible and L-IC$_1$ is added back in to the problem, there is an underlying effect from L-IC$_2$ that tends to pull towards making higher actions more profitable.

Expected utility from a fixed-wage contract paying $CE_{-V_2}(a_1, a_2)$ is

$$\overline{U}(a_1, a_2) = V(CE_{-V_2}(a_1, a_2), a_2) - c(a_1, a_2), \tag{14}$$

when the agent takes action (a_1, a_2). Next, let

$$T(\overline{u}) = \{(a_1, a_2) \in [\underline{a}_1, \overline{a}_1] \times [\underline{a}_2, \overline{a}_2] | \overline{U}(a_1, a_2) \geq \overline{u}\}$$

describe the set of actions for which the agent would voluntarily sign the contract that consists of a fixed wage of $CE_{-V_2}(a_1, a_2)$ and a required level of effort on the job of a_1. The fact that $CE_{-V_2}(a_1, a_2)$ is strictly decreasing in both arguments can be shown to imply that $\overline{U}(a_1, a_2)$ is strictly decreasing in both arguments. This is turn implies that $T(\overline{u})$ is a "decreasing set"; if some action is in the set then all smaller actions are also in the set. Since $T(\overline{u})$ is a decreasing set, the cheapest action is on the boundary of the set when a_1 is contractible. Likewise, the set grows bigger when $\overline{u}$ declines.

Proposition 4 *Assume A1–A5 hold and that $CE_{-V_2}(a_1, a_2)$ as defined in (13) exists. Then $\overline{U}(a_1, a_2)$ is strictly decreasing in a_1 and a_2. Moreover, $T(\overline{u})$ is a decreasing set: If $(a_1, a_2) \geq (a_1', a_2')$ and $(a_1, a_2) \in T(\overline{u})$ then $(a_1', a_2') \in T(\overline{u})$. Finally, if $\overline{u} > \overline{u}'$ then $T(\overline{u}) \subseteq T(\overline{u}')$.*

Proof. See the Appendix. ■

5.3 Redundant participation constraint

It is to be expected that there is an intimate link between L-IC$_2$ and (P) even when a_1 is not contractible. To lay bare this relationship, it turns out to be useful to ask whether an agent with utility function $V(w, a_2)$ is more or less risk averse than an agent with utility function $[-V_2(w, a_2)]$. Comparing the levels of absolute risk aversion with respect to w yields

$$\frac{-\left[-V_{112}(w,a_2)\right]}{\left[-V_{12}(w,a_2)\right]}-\frac{-V_{11}(w,a_2)}{V_1(w,a_2)}\geq 0$$

by (9) and the fact that $V_{12}(w,a_2)<0$. Thus, an agent with utility function $[-V_2(w,a_2)]$ is more risk averse with respect to w than an agent with utility function $V(w,a_2)$. Then, the certainty equivalent of any contract is smaller for the former than for the latter. Even though many different contracts may implement the same action, they all share the same $CE_{-V_2}(a_1,a_2)$ value as determined from L-IC$_2$. Then, since an agent with utility function $V(w,a_2)$ has a larger certainly equivalent, it holds that

$$\int V(w(x_1),a_2)g^1(x_1|a_1)dx_1-c(a_1,a_2)\geq V(CE_{-V_2}(a_1,a_2),a_2)-c(a_1,a_2)\equiv\overline{U}(a_1,a_2).$$

Since $CE_{-V_2}(a_1,a_2)$ is predetermined, $\overline{U}(a_1,a_2)$ is also predetermined. Holding the action fixed, the agent's expected utility thus exceeds $\overline{u}$ if $\overline{u}$ is small enough, $\overline{u}\leq\overline{U}(a_1,a_2)$. In this case, the participation constraint is automatically satisfied. Stated differently, if $\overline{u}$ is small then (P) is redundant – it is implied by L-IC$_2$.[13] Thus, the agent is guaranteed rent above his reservation utility.

Proposition 5 *Assume A1–A5 hold and fix any interior action that is to be induced. Then, the participation constraint is redundant if $\overline{u}$ is small enough. Conversely, if $\overline{u}$ is held fixed then the participation constraint is redundant for any interior action in $T(\overline{u})$.*

Proof. The first part is in the text. The second part is by definition of $T(\overline{u})$. ■

Laffont and Martimort (2002, Section 5.3) present a single-task principal-agent model without private rewards. However, the utility function is not separable in income and effort. They show that the optimal contract may leave the participation constraint slack. This is by choice, however. The principal could make the participation constraint bind, yet he elects not to do so. In contrast, when $\overline{u}$ is small in the current model, it is impossible to make the participation

[13] Note that the argument does not rely on the validity of the FOA. It only relies on L-IC$_2$, which is merely a necessary condition for implementing an interior action.

constraint bind for interior actions. Laffont and Martimort's (2002) model is motivated by the idea that if an agent in a low-income country is guaranteed a certain minimum income then he is able to fulfill basic nutritional needs. Being healthier, the marginal cost of effort decreases. Hence, less powerful incentives are required. The current model instead better describes the dilemmas faced by an agent in a high-income country. Thus, the models and underlying mechanisms are quite different, but non-separability plays a large role in both.

The agent has only two actions available to him in Laffont and Martimort's (2002) model. Alvi (1997) justifies the FOA in a fairly similar model but with a continuous action. He does not discuss the participation constraint in any detail, but he does note that non-separability tends to make the contract flatter.

Generally, some or all actions outside $T(\overline{u})$ can be implemented as well. However, it is interesting to note that this is not always the case. Consider the following specialized version of the model.

Definition (The multiplicative model): In the *multiplicative model*, the rewards function takes the form

$$v(w, x_2) = -m(w)n(x_2), \tag{15}$$

where m and n are strictly *negative* functions that are strictly increasing and strictly concave.

The parts of Assumption A4 that pertain to the rewards function are satisfied in the multiplicative model.[14] The model's special feature is that the inequality in (8) in A5 holds with equality, as does the inequality in (9). In other words, $V(w, a_2)$ and $-V_2(w, a_2)$ are equally risk averse over labor income. Consequently, $CE_V = CE_{-V_2}$ for any contract. Hence, any interior action that is outside $T(\overline{u})$ cannot satisfy L-IC$_2$ and (P) at the same time. In other words, such an action cannot be implemented. This argument can be extended to actions with $a_2 = \overline{a}_2$.

Proposition 6 *In the multiplicative model, any action with $a_2 > \underline{a}_2$ can be implemented if and only if it is in $T(\overline{u})$.*

[14] The term "multiplicative" may invoke thoughts of complementarity rather than substitutability. However, note that the product of the two (negative) functions is multiplied by -1. For this reason, w_1 and w_2 are substitutes.

Proof. See the Appendix. ■

The multiplicative model is of special interest when compared to the LEN model due to Holmström and Milgrom (1987, 1991). This model assumes that the agent has constant absolute risk aversion over total income. Adapting this to the present model, utility from total income is $v(w, x_2) = -e^{-r(w+x_2)}$ if the private rewards, x_2, is monetary, where $r > 0$ measures the degree of risk aversion. However, this fits with the multiplicative model, when $m(w) = -e^{-rw}$ and $n(x_2) = -e^{-rx_2}$. Of course, the two models differ in how much structure is exogenously imposed on the contract and the signal distribution; see Section 7.

5.4 The reduced problem and the FOA

An immediate implication of Proposition 4 is that if the principal is considering the cheapest way to induce some interior action in $T(\overline{u})$ then he can simply ignore the participation constraint. This suggests that the relaxed problem can be further reduced. Thus, consider the *reduced problem*

$$\max_{a_1,a_2,w} B(a_1, a_2) - \int w(x_1)g_1(x_1|a_1)dx_1$$

$$EU_i(a_1, a_2) = 0, \quad i = 1, 2. \tag{L-IC}$$

Note that any solution to the reduced problem that involves an action in $T(\overline{u})$ also solves the relaxed problem because it satisfies all the constraints in the relaxed problem. Moreover, any contract that solves the reduced problem is regular (see the proof of the next result). The structure of the contract is as in (10), with $\lambda = 0$ and $\mu_1 > 0 > \mu_2$. Hence, the contract is incentive compatible, by Lemma 1, and thus satisfies all the constraints of the unrelaxed problem.

Imagine that the second-best action is interior. If the reduced problem identifies an optimal action in the interior of $T(\overline{u})$ then that action correctly identifies the second-best. Moreover, all actions belong to $T(\overline{u})$ if $\overline{u}$ is sufficiently low. In this case, the reduced problem solves the unrelaxed problem. As mentioned, since (P) is slack, the relaxed problem produces the same solution as the reduced problem. Hence, the FOA is valid.

Theorem 2 *Assume that A1–A5 and P1 holds. Moreover, assume that $\overline{u}$ is so* *low that $T(\overline{u}) = \times_{i=1}^{2} [\underline{a}_i, \overline{a}_i]$ and that any second-best action is interior. Then, the reduced and relaxed problems solve the unrelaxed problem and the FOA is valid. Finally, $\lambda = 0$ and $\mu_1 > 0 > \mu_2$.*

Proof. See the Appendix. ■

Next, assume that $T(\overline{u})$ does not contain all actions. First, it is clear from Proposition 6 that the FOA is valid in the multiplicative model since any interior action not in $T(\overline{u})$ does not satisfy the constraints in the relaxed problem. An alternative strategy in the multiplicative model is to solve the reduced problem on the feasible set of actions, $T(\overline{u})$.

More generally, a solution of the reduced problem that falls outside $T(\overline{u})$ is optimal if it satisfies the participation constraint. In this case, the relaxed problem would identify the same solution as the reduced problem. The implication is that if the reduced and relaxed problems has a solution that coincides, then that solution solves the unrelaxed problem and is optimal.

Proposition 7 *Assume that A1–A5 and P1 hold and that any second-best action is interior. Then, any solution of the reduced and relaxed problems that coincide solves the unrelaxed problem. Any optimal contract identified in this manner is regular, with $\lambda = 0$ and $\mu_1 > 0 > \mu_2$.*

Proof. See the Appendix. ■

Three remarks on these results follow. First, imagine that there is a unique second-best action, which is in the interior of $T(\overline{u})$. Then, a marginal change in $\overline{u}$ does not change the cost of implementing that action. Hence, the optimal contract is unaffected by a small change in reservation utility. This never occurs in the standard model, where (P) always binds. In this sense, the model is consistent with contracts that are less sensitive to shocks affecting labor supply.

Second, note that the part of Assumption A4 that imposes $c_{12} \geq 0$ is used in Lemma 4 and Proposition 4 to structure the set $T(\overline{u})$, but is not used in the proofs of Theorem 2 and Proposition 5. Hence, these results are valid without the $c_{12} \geq 0$ assumption. However, any comparative statics that make use of e.g. Lemma 4 is potentially sensitive to this assumption.

Third, there is a subtle difference between the proofs of Theorems 1 and 2. Theorem 1 makes use of (12) but not (11). Thus, the proof implies that for any interior and possibly inoptimal a_1, the optimal accompanying a_2 is implemented in the cheapest possible manner with a FOA contract like the one in (10). Theorem 2 makes use of neither (11) or (12). Thus, it implies that any interior and possibly inoptimal (a_1, a_2) is most cheaply implemented with a FOA contract. Hence, Theorem 2 can be used to ask if implementation costs are monotonic in a_1 and a_2 (see the example in Section 6.1). It also implies that the effects in Proposition 3 are present for any interior action, not only the second-best action.

5.5 Intrinsic motivation revisited

Since the optimal contract is regular if the second-best action is in the interior of $T(\overline{u})$, Lemma 1 implies that $EU_{12}(a_1, a_2) < 0$. That is, the tasks are strict substitutes. This property is exploited in the following thought experiment.

Consider a principal who hires different but identical agents in different countries. The principal's benefit function is different in these various countries, such that the second-best action varies with the country. Consider an admittedly extreme case where it is optimal for the principal to induce the exact same a_1 but different a_2's in different countries. Assume that all these actions are in the interior of $T(\overline{u})$. Assume also that the outsider can observe a_1.

As the a_2's differ across countries, the outsider would observe qualitatively different contracts inducing the exact same effort on the job. This would appear puzzling because agents are identical and the standard model predicts that there is a unique optimal contract for any given a_1. Imagine nevertheless that the outsider is determined to use the variety of contracts to learn about a perceived trade-off between intrinsic and extrinsic motivation.

Consider two countries, with second-best action (a_1^*, a_2') and (a_1^*, a_2''), respectively, with $a_2' < a_2''$. Let ω' and ω'' denote the corresponding contracts. Given contract ω, write expected utility as $EU(a_1, a_2|\omega)$. Assume that the outsider believes a_2 is exogenously fixed at some value that is estimated to be between a_2' and a_2''. Now, L-IC$_2$ and $EU_{12} < 0$ imply that for any such estimate, $a_2 \in [a_2', a_2'']$,

$$EU_1(a_1^*, a_2|\omega'') \geq EU_1(a_1^*, a_2''|\omega'') = 0 = EU_1(a_1^*, a_2'|\omega') \geq EU_1(a_1^*, a_2|\omega'),$$

with at least one strict inequality. As suggested by Proposition 2, the agent has "stronger incentives" on the job when he is induced to work harder at home.

The next part of the argument is that the outsider believes that the contract ω' gives greater utility to the agent than contract ω'', given action (a_1^*, a_2). This can be established formally in the multiplicative model, where $v(w, x_2) = -m(w)n(x_2)$. Since the outsider believes that a_2, and thus the distribution of x_2, is fixed, the part of expected utility that involves the expectation of $n(x_2)$ is thought to be the same regardless of the contract. On the other hand, it is observed that the expectation of $m(w)$ is be greater under ω' than ω''. The reason is that higher utility from labor income is required to entice the agent to take less of an interest in pursuing private rewards, such that $a_2' < a_2''$ follows from L-IC$_2$.

Thus, in the multiplicative model, for any estimated $a_2 \in [a_2', a_2'']$, the outsider concludes that

$$EU_1(a_1^*, a_2|\omega'') > EU_1(a_1^*, a_2|\omega') \text{ and } EU(a_1^*, a_2|\omega'') < EU(a_1^*, a_2|\omega').$$

One interpretation that comes to mind is that ω' provides weaker extrinsic motivation (the return to higher effort on the job is lower) but higher intrinsic motivation (derived from higher rent) than ω''. In short, the outsider may conclude that there is a trade-off between the two.

Assume next that the optimal level of a_1 is different in different countries, due to differences in $B(a_1, a_2)$. Focusing on the multiplicative model, let

$$M(a_1|\omega) = \int m(w(x_1))g^1(x_1|a_1)dx_1$$

denote the expected utility from labor income and

$$M'(a_1|\omega) = \int m(w(x_1))g^1_{a_1}(x_1|a_1)dx_1$$

denote the increase in utility from labor income that comes from a marginal increase in effort on the job. The conceptual advantage of the multiplicative model is that if the outsider ignores a_2 or believes that it is exogenously fixed, then $M(a_1|\omega)$ and $M'(a_1|\omega)$ appear to the outsider to be clean measures of labor rewards and incentives, respectively. However, in the following setting, it is optimal to induce lower a_2 the higher the desired level of a_1 is. Recall that when a_2 is low, the agent's rewards come primarily from work and so flatter incentives can incentivize him. This effect, if overlooked by the outsider, is enough to give the illusion that the agent is intrinsically motivated by higher labor rewards.

Proposition 8 *Assume that A1–A4, P1, and (15) hold. Assume that* $t(a_1) = \max\{a_2|(a_1, a_2) \in T(\overline{u})\}$ *is interior for all* a_1*. Finally, assume that* $G^2(x_2|a_2)$ *is log-convex in* a_2 *and that* $c(a_1, a_2) = c_1a_1 + c_2a_2$ *and* $B_2(a_1, a_2) = 0$*. Then, the cheapest way to implement any* a_1 *is to induce* $a_2 = t(a_1)$ *at the same time. Let* ω'' *and* ω' *denote the optimal contracts that induce* $(a_1'', t(a_1''))$ *and* $(a_1', t(a_1'))$*, respectively. Then, if* $a_1'' > a_1'$ *are both interior,*

$$M(a_1'|\omega') < M(a_1''|\omega'') \text{ and } M'(a_1'|\omega') > M'(a_1''|\omega''). \tag{16}$$

Proof. The proof is lengthy and involves several steps. It combines Proposition 1 and Corollary 1 in Kirkegaard (2016), to which the reader is referred. ■

Thus, the agent earns higher utility from labor income the harder he is induced to work on the job. However, an outsider who fails to take into account that a_2 is endogenous would conclude that the marginal return to extra effort is *lower* the harder the agent is induced to work. In other words, it looks as if the agent works harder when given weaker explicit incentives but higher labor rewards. Log-convexity is discussed in Section 7.

5.6 Boundary actions and three job strata

The analysis thus far has focused on interior a_2. If $\overline{a}_2$ is to be induced, incentive compatibility requires that $EU_2(a_1, \overline{a}_2) \geq 0$, or

$$\int [-V_2(w(x_1), \overline{a}_2)]\, g^1(x_1|a_1)dx_1 \leq -c_2(a_1, \overline{a}_2).$$

Thus, incentive compatibility only provides an upper bound on CE_{-V_2}. Hence, the principal has more degrees of freedom to meet the participation constraint. For this reason, implementation costs may drop discontinuously at $\overline{a}_2$. In fact, if $(a_1, \overline{a}_2) \in T(\overline{u})$ then any contract that satisfies L-IC$_1$ and makes (P) bind at action $(a_1, \overline{a}_2)$ automatically satisfies $EU_2(a_1, \overline{a}_2) \geq 0$, generally with a strict inequality. Hence, the optimal contract that induces extreme effort in pursuit of private rewards exactly mimics a standard contract, with $\lambda > 0$ but $\mu_2 = 0$.[15]

Theorem 2 assumes that the second-best action is interior. In fact, $a_2 = \overline{a}_2$ cannot be optimal if $B(a_1, a_2)$ decreases sufficiently quickly in a_2 around $\overline{a}_2$, even though implementation costs drop discontinuously at $\overline{a}_2$. This observation creates a stronger tie between Theorems 1 and 2, the former of which assumes $B_2 \leq 0$. Note also that it may not be possible to implement $a_2 = \overline{a}_2$, by Proposition 6.[16]

The discontinuity is perhaps not surprising given that the standard model also has a discontinuity. In particular, in both the standard and the current model, implementation costs are discontinuous in a_1 at $\underline{a}_1$. Here, however, the discontinuity in a_2 at $\overline{a}_2$ have more interesting economic implications, as follows.

Imagine that $\overline{u}$ is very low, such that $T(\overline{u})$ consists of all actions. Then, implementation costs are not affected by a further decrease in $\overline{u}$ for actions in the interior but they diminish for actions on the $a_2 = \overline{a}_2$ boundary. Thus, when $\overline{u}$ is sufficiently low, the second-best action must feature $a_2 = \overline{a}_2$, such that the principal can benefit from making the participation constraint bind.

In comparison, assume that $\overline{u}$ is very large, such that $T(\overline{u})$ is empty. In the multiplicative model, Proposition 6 implies that only $a_2 = \underline{a}_2$ can be implemented. In this case, it can be verified that the constraint that $EU_2 \leq 0$ is met when the participation constraint binds. Thus, in the multiplicative model, at least, (P) binds if reservation utility is either very high or very low. Moreover, in these cases the incentive constraint on a_2 is implied by (P). Thus, it is only if $\overline{u}$ is in a middle range that the agent may achieve rent in excess of the reservation utility.

In conclusion, there are three strata of jobs, depending on the agent's reservation utility. Agents with extreme reservation utilities exert either extremely high or extremely low effort in pursuing a rewarding outside life. An agent with a moderate reservation utility is instead manipulated to exert moderate effort along this dimension. See also the example in Section 6.1.

[15] The arguments in Section 3 apply. If the agent is already setting $a_2 = \overline{a}_2$ then he cannot increase a_2 further to substitute for lower wages. Thus, the incentives with respect to a_1 are unchanged when $w(x_1)$ is replaced by $\widehat{w}_\varepsilon(x_1)$ as defined in Proposition 1. Hence, (P) binds.

[16] Outside the multiplicative model, even if action $(a_1, \overline{a}_2) \notin T(\overline{u})$ is implementably then it may be prohibitively costly to implement if $c(a_1, \overline{a}_2)$ is very large.

6 A reinterpretation of the reduced problem

Only L-IC$_1$ and L-IC$_2$ enter the reduced problem. Although these are equality constraint, for the sake of argument imagine weakening L-IC$_2$ by turning it into an inequality constraint, such that the constraints can be written

$$\int V(w(x_1), a_2) g^1_{a_1}(x_1|a_1)dx_1 - c_1(a_1, a_2) \;=\; 0 \tag{17}$$

$$\int [-V_2(w(x_1), a_2)]\, g^1(x_1|a_1)dx_1 - c(a_1, a_2) \;\geq\; -c(a_1, a_2) - c_2(a_1, a_2) \tag{18}$$

and consider how an outside observer would interpret these conditions. First, the outsider believes a_2 is fixed and that the agent's only choice variable is a_1. Then, the two constraints are similar to the constraints that enter a standard moral hazard problem where a_1 is incentivized. The first constraint provides the correct incentives on the job for an agent with utility function $V(w(x_1), a_2)$. The second constraint resembles a participation constraint for someone with utility function $[-V_2(w(x_1), a_2)]$, with one important difference. Specifically, the right hand side – which represents "reservation utility" in the analogy – is strictly decreasing in a_1. Thus, it is as if participation is decided by a different side of the agent's personality which (i) is more risk averse and which (ii) experiences some kind of self-satisfaction from working hard, thus lowering the threshold for participation when the agent expects to be working hard. Since this side of the agent's personality is more risk averse, it is more put off by any risk included in the contract. On the other hand, it is intrinsically motivated to work hard.

To cement the analogy, note that the standard argument described in the second half of Section 3.1 can easily be used to prove that the "participation constraint" must bind. Hence, the inequality constraint effectively becomes an equality constraint, as in the reduced problem.[17] Thus, consider the outsider who believes that a_2 is fixed at some interior a_2' and who wishes to induce some interior a_1. Using the above incentive compatibility and participation constraint then identifies the correct solution if that solution is in $T(\overline{u})$ or if the "real" participation constraint is satisfied. In particular, the agent optimally responds

[17] The inequality constraint can be written as $EU_2(a_1, a_2) \leq 0$, implying that $\mu_2 \leq 0$. Hence, in the reduced problem, $\mu_2 \leq 0$ is the natural counterpart to $\lambda \geq 0$ in a standard problem.

by choosing $a_2 = a_2'$, thus confirming the outsider's belief that a_2 is fixed.

It is useful to think more carefully about the two characteristics, (i) and (ii), of the personality that evaluates participation in the reduced problem. For a fixed action, the first property implies that the contract looks different from a standard contract where participation is determined by the same personality that evaluates incentives. In particular, as discussed in Section 4.3, property (i) tends to make the contract flatter. The agent who decides on participation is more risk averse, and so it becomes more expensive to make him accept steeper incentives.

The second property, (ii), implies that there is an indirect benefit to the principal of asking the agent to work harder on the job. Since this personality experiences self-satisfaction from working hard, employment compensation does not need to be increased as much to induce higher effort as would otherwise have been the case. It is "as if" reservation utility decreases when the agent is asked to work harder on the job. If this effect is strong enough, implementation costs may be decreasing in a_1. See Section 6.1 for an example.

It also follows that an outside observer who understands property (i) but not property (ii) will underestimate the benefit from increasing a_1. In terms of the direct and indirect effects in Proposition 3, the outsider does not take the full cost savings from the indirect effect into account. Hence, the outsider erroneously believes that the agent is induced to work too hard on the job, and that the principal would benefit from working the agent less hard.

6.1 An example

Assume in this subsection that

$$G^i(x_i|a_i) = \left(1 - e^{-a_i}\right) x_i^2 + \left(e^{-a_i}\right) x_i, \quad x_i \in [0,1]$$

and $a_i \in \left[\frac{1}{9}, \frac{1}{8}\right]$, $i = 1, 2$. It can be verified that Assumptions A1–A3 are satisfied. Assume also that

$$c(a_1, a_2) = \left(a_1 - \frac{1}{8}\right) + \frac{6}{5}\left(a_2 - \frac{1}{8}\right) + \frac{64\left(a_1 - \frac{1}{8}\right)^2 + 16\left(a_1 - \frac{1}{8}\right)\left(a_2 - \frac{1}{8}\right) + \left(a_2 - \frac{1}{8}\right)^2}{2}.$$

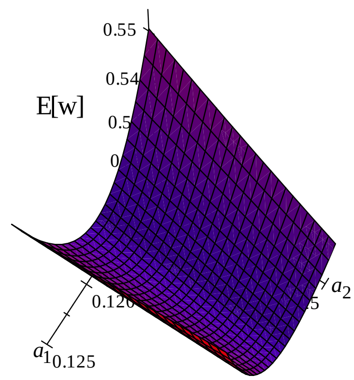

Figure 1: Implementation costs.

The rewards function is multiplicative, with

$$v(w, x_2) = -\left(2\sqrt{w} - k\right)\left(\frac{15\sqrt{x_2} - 16}{2}\right),$$

where $w \in [0, (\frac{k}{2})^2]$, $x_2 \in [0, 1]$ and $k = 2.6851$. Assumption A4 is satisfied, with the exception that w cannot be any real number. However, k is selected to ensure that any optimal contract features wages in the interior and that $m(w) = 2\sqrt{w} - k$ is negative as required in the multiplicative model.

Given the rewards function is separable, expected utility can be reduced to

$$EU(a_1, a_2) = N(a_2)\int_0^1 \left(2\sqrt{w(x_1)} - k\right) g^1(x_1|a_1)dx_1 - c(a_1, a_2),$$

where $N(a_2) = 2 + e^{-a_2} > 0$. Proposition 6 implies that attention can be restricted to actions in $T(\overline{u})$, where (P) is automatically satisfied. Then, any contract derived from the reduced problem must satisfy

$$\sqrt{w} = \mu_2 N'(a_2) + \mu_1 N(a_2) l^1_{a_1}(x_1|a_1)$$

or, by renaming variables,

$$\sqrt{w} = \widehat{\lambda} + \widehat{\mu} l^1_{a_1}(x_1|a_1), \tag{19}$$

which takes the same form as in a standard problem. Utilizing (19), it is then possible to use similar steps as in Jewitt et al (2008, Example 1) or Kirkegaard (2017) to explicitly derive $\widehat{\lambda}$ and $\widehat{\mu}$. In short, the contract can be characterized in closed form and implementation costs can be derived. Details are available on request. Implementation costs are depicted in Figure 1 for any interior action, assuming that all actions can be implemented.

Figure 2 sketches three iso-cost curves in (a_1, a_2) space, with implementation costs increasing as one moves away from a minimum point at $(a_1, a_2) = (0.1162, \frac{1}{8})$. Although it is hard to see, the eastern-most curve is not vertical but instead "bends inwards". For the remainder of this discussion, assume that $B(a_1, a_2)$ is independent in a_2. An iso-benefit curve is then a vertical line in Figure 2. The point indicated in Figure 2 is the tangency point between such an iso-benefit curve and the iso-cost curve. To continue, assume also that reservation utility is $\overline{u} = -3.85$. Figure 2 also depicts the set $T(\overline{u})$. The thick curve describes the boundary of $T(\overline{u})$, implying that $T(\overline{u})$ consists of all actions below this curve. Note that the shape is consistent with Proposition 4. By Proposition 6, only actions in this set can be implemented.

Holding a_2 fixed, implementation costs are U-shaped in a_1 (Figure 1). The fact that costs may be locally decreasing in a_1 reflects the tension between the direct and indirect effects discussed around Proposition 3. In the reinterpreted problem, the trade-off is between providing steeper incentives but providing those to an agent who is happier to work when he is asked to work hard.

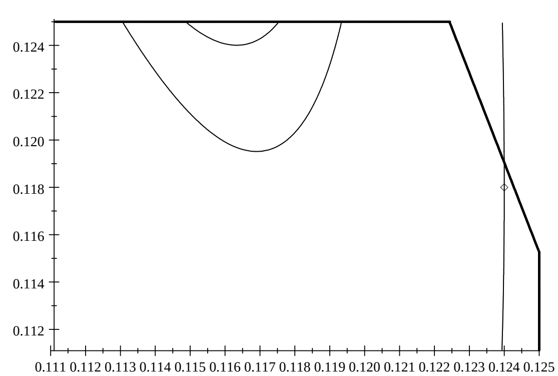

Figure 2: Feasible set & iso-cost curves.

Next, consider holding a_1 fixed but varying a_2. Proposition 3 signifies that there is a tension between the direct and indirect effects of changing a_2 and it is not a priori clear which one dominates. In this example, if $a_1 = \overline{a}_1$ then costs are globally increasing in a_2. Thus, if the principal's benefit function $B(a_1, a_2)$ is rapidly increasing in a_1 then the second best action is $(\overline{a}_1, \underline{a}_2) \in T(\overline{u})$.[18] In this case, the agent "puts all his eggs in one basket" and focuses all his efforts on excelling at work. The agent experiences large utility from labor income, making him disinterested in outside rewards. Conversely, since he is not seeking outside rewards he does not need strong incentives to work hard on the job. Thus, the contract carries little risk, which in turn makes it cheaper to give the agent high utility from labor income.

Consider now a_1 close to $\underline{a}_1$. Here, costs are globally decreasing in a_2. However, if $B(a_1, a_2)$ is increasing in a_1 then it can never be optimal to induce an a_1 that is small but strictly higher than $\underline{a}_1$. The reason is that implementation costs are decreasing and benefits increasing in a_1 when a_1 is small. However, there is a discontinuity in costs at $a_1 = \underline{a}_1$, which can be implemented with a flat wage. By Lemma 4, the cheapest level of a_2 to accompany $\underline{a}_1$ is $a_2 = \overline{a}_2$. The cost of implementing $(\underline{a}_1, \overline{a}_2)$ with a flat wage is 0.4534, which is far lower than the cost of implementing any action with $a_1 > \underline{a}_1$. Thus, if the benefit function does not increase too fast with a_1, then the second-best action is $(\underline{a}_1, \overline{a}_2)$.

Thus, the second-best action is $(\underline{a}_1, \overline{a}_2)$ when $B(a_1, a_2)$ is flat but $(\overline{a}_1, \underline{a}_2)$ when $B(a_1, a_2)$ is very steep. In either case, the agent puts all his eggs in one basket, focusing all his efforts on one particular activity. Next, assume that $B(a_1, a_2)$ is rapidly increasing in a_1 until about $a_1 = 0.124$, after which it becomes almost completely flat in a_1. Then, the second-best action is obtained at a point of tangency between the iso-cost and iso-benefit functions. The point marked in Figure 2 is at $(a_1, a_2) = (0.124, 0.11792)$. Thus, the second-best is in the interior of $T(\overline{u})$. Note that the participation constraint is slack such that the agent earns more than his reservation utility. Here, the agent's day is more balanced between work and life outside the office.

[18] At this corner of the action space, implementation costs can be shown to be continuous. Recall that $(\overline{a}_1, \overline{a}_2)$ cannot be implemented because it is outside $T(\overline{u})$.

This discussion reveals three "strata" of jobs, depending on the features of the benefit function. They range from complete devotion to the job to complete indifference, with an intermediate, more balanced, job type in the middle. Of course, the standard model can also generate different levels of a_1 by varying the benefit function, but it ignores what happens after the office closes. Likewise, it cannot generate rent in excess of reservation utility.

6.2 Three reasons for higher wages

Expected wages are higher in this model than in the standard model for three reasons. First, it is expensive to compensate the $-V_2$ personality for the risk needed to incentivize the V personality (property (i) in the reinterpreted problem). Second, the agent's expected utility may exceed reservation utility (Theorem 2 and the example in Section 6.1). Finally, the agent works too hard on the job, given a_2 (property (ii)).

7 Discussion

This section discusses Assumptions A1–A5 in more detail. Examples are given in which A4 and A5 do not hold. A discussion of common agency follows.

7.1 Assumptions A1–A2

Assumption A1 (independence) assumes that the signal x_1 and the private reward x_2 are independent. For example, there is little reason to think that job performance and the mastery of a hobby are correlated. In other settings, such as when the agent is moonlighting in the same industry, the independence assumption is harder to justify. However, the assumption may have some behavioral justification even in such cases. In particular, there is a growing literature on the prevalence and consequences of *correlation neglect*. See e.g. Levy and Razin (2015) and the references therein. In the current context, correlation neglect arises if the principal and the agent know the marginal distributions, but ignore any correlation between the random variables in the joint distribution.

There are at least two technical problems related to relaxing the independence assumption. One is to establish a counterpart to Lemma 1 for "well-behaved" contracts. However, the FOA contract changes, meaning that (10) may no longer apply. Thus, it also becomes harder to verify whether the contract is "well-behaved" in the first place. In short, the independence assumption captures the main price of allowing the rewards function to be non-separable. Example 1 in Section 7.3 discusses separable rewards functions.

Assumption A2 (MLRP) ensures both that (i) the contract is regular and that (ii) a first order stochastic dominance property holds, i.e. that $G^i_{a_i}(x_i|a_i) < 0$ for $x_i \in (\underline{x}_i, \overline{x}_i)$. The assumption that $g^1(x_1|a_1)$ is log-supermodular can be replaced with the assumption that the first order stochastic dominance property holds and that there is an exogenous restriction that the contract must be non-decreasing in x_1. Such a restriction arises if the agent can sabotage the signal after it has been realized but before the principal observes it.

The assumption that $g^2(x_2|a_2)$ is log-supermodular plays a role in the aggregation result in Lemma 2. It can be replaced by the first order stochastic dominance property and the more direct assumption that (9) holds. Note that (9) is automatic in the multiplicative model.

7.2 Assumption A3

Assumption A3 (LOCC) is a technically motivated assumption that is instrumental in justifying the FOA. It is a direct extension of Rogerson's (1985) convexity assumption. Recall that Rogerson assumes that there is a single signal and a single task. Conlon (2009) presents justifications of the FOA that permit multiple signals but a single task. Kirkegaard (2017) allows multiple tasks, under the assumption that A1 holds. However, private rewards are ruled out and all signals are contractible. In the previous version of the current paper, Kirkegaard (2016), justifications of the FOA with private rewards are given that are in the spirit of Jewitt's (1988) single-task justifications.

A sufficient condition for LOCC is that G^1 and G^2 are both log-convex. The product of log-convex functions is itself log-convex, and therefore necessarily convex. Alternatively, fix some G^1 that is strictly convex in a_1, but not necessarily log-convex. Then, there is always some "sufficiently convex" G^2 function that ensures that Assumption A3 is satisfied. For example, a non-negative function $h(z)$ is said to be ρ-convex if $h(z)^\rho/\rho$ is convex, or $h''(z)h(z)/h'(z)^2 \geq 1-\rho$ for all z. Thus, a ρ-convex function is log-convex if and only if $\rho \leq 0$ (and convex if and only if $\rho \leq 1$). It is easy to see that if $G^2(x_2|a_2)$ satisfies Assumption A2 and is ρ-convex in a_2 (for all x_2) for some small enough ρ (i.e. ρ is negative, but numerically large), then Assumption A3 is satisfied.[19] To reiterate, as long as G^1 satisfies a strict version of CDFC there are G^2 functions that will permit the FOA to be justified even when allowing for private rewards.

There are some similarities between the current model of private rewards and the literature on hidden savings. Ábrahám et al (2011) consider a situation where the agent works for the principal while simultaneously privately investing in a risk-free asset. There is thus no uncertainty concerning the return to the non-contractible action. Hence, performance on the job, x_1, is the only source of uncertainty. Ábrahám et al (2011) justify the FOA by assuming that the distribution of x_1 is log-convex in effort on the job, a_1, and that the agent has decreasing absolute risk aversion. Assumptions A3 (LOCC) and A5 (log-supermodularity) in the current paper can be seen as extensions that allow returns that are both stochastic and potentially non-monetary.

More specifically, let a_2 denote the dollar amount that the agent saves. Savings has a risk-free rate of return of r. Letting $U(\cdot)$ denote the Bernoulli utility function over total wealth, the agent's utility upon earning $w(x_1)$ on the job and ra_2 from savings is $U(w(x_1)+ra_2)$. Given action (a_1, a_2), integration by parts yields expected utility from rewards of

$$\int U(w(x_1)+ra_2)g^1(x_1|a_1)dx_1 = U(w(\overline{x}_1)+ra_2) - \int U'(w(x_1)+ra_2)w'(x_1)G^1(x_1|a_1)dx_1. \tag{20}$$

The first term is concave in (a_1, a_2), given the agent is risk averse. Next, note that decreasing absolute risk aversion in total income is equivalent to log-convexity of $U'(\cdot)$. First, since $U'(\cdot)$ is log-convex in w, the right hand side of the counterpart

[19] The inequality in footnote 4 can be written $G^1G^1_{a_1a_1}(G^2G^2_{a_2a_2}/\left(G^2_{a_2}\right)^2) - \left(G^1_{a_1}\right)^2 \geq 0$, for interior (x_1, x_2). By ρ-convexity, the left hand side is greater than $G^1G^1_{a_1a_1}(1-\rho) - \left(G^1_{a_1}\right)^2 \geq 0$. Hence, the inequality is satisfied if ρ is small enough.

to (10) is then well-behaved. Second, $U'(\cdot)$ is log-convex in a_2. Then, assuming G^1 is log-convex in a_1, the integrand in the above expression is now the product of functions that are log-convex in (a_1, a_2). Hence, the integrand is log-convex and therefore convex in (a_1, a_2). It now follows that expected utility from rewards are concave in the agent's action. These are the main steps in Ábrahám et al's (2011) justification of the FOA.

Note that log-convexity of $U'(\cdot)$ plays two roles above. Moreover, log-convexity of $U'(\cdot)$ is equivalent to $U'(\cdot)$ being log-supermodular in (w, a_2). In the current paper, $V_1(w, a_2)$ plays the role of $U'(\cdot)$ in (20). Assumption A5 implies that $V_1(w, a_2)$ is log-supermodular in (w, a_2) (Lemma 2). This assumption is used to discipline the FOA contract in (10). However, since $V_1(w, a_2)$ is not necessarily log-convex in a_2, the above argument cannot be used to establish concavity. Instead, concavity in Lemma 1 comes from the convexity assumption in Assumption A3 (LOCC) and the substitutability assumption that $v_{12} < 0$. In A3, convexity also reduces to requiring that the product of two functions, $G^1(x_1|a_1)$ and $G^2(x_2|a_2)$, are convex in (a_1, a_2). Log-convexity of each function is again sufficient.

7.3 Assumptions A4–A5

The assumption in A4 that $v_{12} < 0$ is important in several places, including early on in establishing concavity of the agent's expected payoff (Lemma 1). Relaxing this assumption to allow rewards from different sources to be complements is an important topic for future research but it is likely to be technically challenging.

Theorem 2 does not require $c_{12} \geq 0$. Neither do the following two examples. In fact, the first example does not even require Assumption A1 (independence). This example illustrates why Assumption A4 rules out $v_{12} = 0$. Specifically, the *additive model* has an additively separable rewards function which eliminates any direct interaction between rewards from different sources. As a result, the model is not substantially different from the standard model. The point is that the paper's new results stem from interdependencies in the rewards function. The additive model also effectively reproduces the results of the Linear-Exponential-Normal (LEN) model.

Example 1 (The Additive Model): Assume that

$$v(w, x_2) = u(w) + q(x_2),$$

where u and q are strictly increasing and strictly concave functions. Note that $v_{12} = 0$. Assume that $c(a_1, a_2)$ is strictly increasing and convex. Recall that a_2 determines the distribution of x_2. Hence, let $Q(a_2)$ denote the expectation of $q(x_2)$, given a_2. By Assumptions A2 and A3, $Q(a_2)$ is strictly increasing and strictly concave. Similarly, a_1 determines the distribution of x_1 and thus the distribution of wages. Let ω denote the contract and write $U(a_1|\omega)$ as the expectation of $u(w(x_1))$, given a_1. Thus,

$$EU(a_1, a_2) = U(a_1|\omega) + Q(a_2) - c(a_1, a_2). \tag{21}$$

Note that for a fixed a_1, the agent's optimal a_2 is unique and independent of the contract. In other words, once the principal has decided which a_1 he wishes to induce, a_2 is predetermined and impossible to manipulate. Henceforth, let $a_2(a_1)$ denote the optimal value of a_2, given a_1. The model is now essentially a standard model since the agent's action is effectively one-dimensional. For concreteness,

$$EU(a_1) = U(a_1|\omega) + Q(a_2(a_1)) - c(a_1, a_2(a_1)).$$

Unsurprisingly, the model has standard features. The principal designs the contract to manipulate a_1. He has to respect the participation constraint that

$$U(a_1|\omega) \geq \overline{u} - Q(a_2(a_1)) + c(a_1, a_2(a_1)).$$

It is easy to verify that the right hand side is increasing in a_1. Thus, the agent must be promised higher rewards from labor income to accept a contract that induces higher effort. To induce interior effort a_1 on the job, L-IC$_1$ is

$$\frac{\partial U(a_1|\omega)}{\partial a_1} = c_1(a_1, a_2(a_1)).$$

Again, it can be checked that the right hand side is increasing in a_1. Thus, to induce higher effort on the job, expected utility from rewards must respond more dramatically to changes in effort. These conclusions are entirely standard.

The LEN model produces identical results. The reason is that the agent's certainty equivalent in the LEN model is separable, as in (21). See Kirkegaard (2016) for a more detailed discussion of private rewards in the LEN model. The chief difference is that the LEN model stipulates that contracts are linear, $w(x_1) = \beta + \alpha x_1$, and that the agent's action is to pick the means of normally distributed signals. Thus, $U(a_1|\omega) = \beta + \alpha a_1$ and $\frac{\partial U(a_1|\omega)}{\partial a_1} = \alpha$. Hence, the LEN model has an extremely convenient one-parameter measure of the strength of incentives, α. The higher α is, the harder the agent works on the job. The current paper does not have an equally convenient measure of incentives. Instead, EU_1 is used as a summary measure of the incentive to work marginally harder.[20] ▲

Assumption A5 is violated in the following model, where utility is quadratic in total income. This gives rise to a rather extreme situation in which (P) is not redundant but the principal has no interest in making it bind.

Example 2 (The Quadratic Model): Assume that the agent's private rewards are monetary and that his utility is quadratic in total income. That is, $v(w, x_2)$ takes the form

$$\begin{aligned} v(w, x_2) &= (w + x_2) - \gamma (w + x_2)^2, \\ &= (w + x_2) - \gamma \left(w^2 + x_2^2\right) - 2\gamma w x_2 \end{aligned}$$

where γ is a strictly positive constant. The domain of $v(w, x_2)$ is a subset of $\mathbb{R}_+^2$ such that the function is strictly increasing and strictly concave in w and in x_2. Clearly, income from the two sources are substitutes. Note, however, that the agent exhibits increasing absolute risk aversion, thus violating Assumption A5.

Let $x(a_2) \geq 0$ denote the expected value of x_2, as a function of a_2. Likewise, let $s(a_2)$ denote the second moment (the expected value of x_2^2) of x_2 as a function of a_2. Both functions are exogenous. By Assumptions A2 and A3, $x(a_2)$ and $s(a_2)$ are strictly increasing and strictly concave. Next, the distribution of x_1 is

[20] In the LEN model, α and β reveals (i) how the contract depends on x_1, which in turn (ii) determines the return to a marginal increase in effort on the job. Measures such as EU_1 bypasses the first part to say something more directly about the second part.

determined by a_1. Thus, given the endogenous contract, let $\overline{w}(a_1) \geq 0$ denote the expected wage as a function of a_1 and let $\overline{s}(a_1)$ denote the second moment. Then, given x_1 and x_2, the agent's expected utility from action (a_1, a_2) is

$$EU(a_1, a_2) = \overline{w}(a_1) + x(a_2) - \gamma\left(\overline{s}(a_1) + s(a_2)\right) - 2\gamma\overline{w}(a_1)x(a_2) - c(a_1, a_2).$$

The local incentive compatibility constraints are

$$\begin{aligned} \overline{w}'(a_1) - \gamma\overline{s}'(a_1) - 2\gamma\overline{w}'(a_1)x(a_2) &= c_1(a_1, a_2) \\ x'(a_2) - \gamma s'(a_2) - 2\gamma\overline{w}(a_1)x'(a_2) &= c_2(a_1, a_2). \end{aligned}$$

The unique feature of the quadratic model comes from L-IC$_2$. The only endogenous element is the expected income from labor, $\overline{w}(a_1)$. Thus, *the expected wage is already pinned down from incentive compatibility.* Note that this is all a risk neutral principal cares about. Thus, there is no particular reason to make the participation constraint binding. Hence, for any *fixed* interior action, there is no unique optimal contract and some optimal contracts leave the agent with rent above his reservation utility. Any optimal contract has the same expected wage but differ in the level of risk (as summarized e.g. by the variance, $\overline{s}(a_1) - \overline{w}(a_1)^2$) imposed on the agent. This property is irreconcilable with the standard model. After all, a basic insight from the standard principal-agent model is that risk should not exceed what is required to provide incentives, since the expected wage must normally increase to compensate. That argument, however, relies on a binding participation constraint.

On the other hand, a fixed a_1 can be implemented with a range of different a_2's, each dictating different expected wage costs. To satisfy L-IC$_2$ when a_2 increases, it is necessary that $\overline{w}(a_1)$ decreases. Consequently, if the principal does not care directly about a_2, or $B_2 = 0$, then he will spur the agent to work as hard as possible on accumulating private rewards. Hence, it is in the principal's interest to entice the agent to have a rewarding home life.[21]

Although $\overline{w}(a_1)$ is determined by the incentive constraint on a_2, the agent's

[21] When $\overline{a}_2$ is to be induced, the constraint is that $EU_2 \geq 0$. This is more likely to be satisfied the lower $\overline{w}(x_1)$ is. Thus, the participation constraint is expected to bind.

marginal costs also depend on a_1. If $c_{12} > 0$, marginal costs with respect to a_2 increases when a_1 increases. To maintain the same level of a_2 the returns to private rewards must be made larger. This is achieved by lowering $\overline{w}(a_1)$. In short, *it is cheaper to induce higher levels of effort on the job*, a_1. Hence, if the agent's effort is productive, or $B_1 > 0$, it is optimal for the principal to induce the agent to work as hard as possible on the job. The reason is, again, that it is only the incentive constraint on a_2 that determines contracting costs. ▲

Finally, recall that Assumption A4 rules out a binding minimum wage. The proof of Theorem 1 is easily extended to allow for minimum wages, but it appears more problematic to extend Theorem 2. Moreover, some actions, e.g. those with small CE_{-V_2}, may not be implementable with a minimum wage in place.

7.4 Common agency

Given a_2, the principal considers the distribution of private rewards to be fixed. However, the outside rewards are sometimes derived from other principal-agent relationships. This is the case when the agent holds several jobs. In such cases of common agency, principals are strategically interacting with each other.

Bernheim and Whinston (1986) were first to consider such situations. However, they assume that every principal observes the same information. Thus, any principal can observe and verify how well the agent performed for other principals. Bernheim and Whinston (1986) establish that the equilibrium action is implemented at a total cost that coincides with the total cost that would have obtained if the principals could collude (or merge). As Bernheim and Whinston (1986) explain: "We can always view a principal as constructing his incentive scheme in two steps: he first undoes what all the other principals have offered and then makes an 'aggregate' offer [...]. Clearly, if we are at an equilibrium, each principal must, in this second step, select an aggregate offer that implements the equilibrium action at minimum cost." On the other hand, competition between principals typically distorts the equilibrium action away from the second-best.

The model in the current paper instead assumes that outside rewards are private. That is, any given principal cannot observe how well the agent performs for another principal. Holmström and Milgrom (1988) use the term "disjoint observations" to refer to such a setting.

Holmström and Milgrom (1988) use the LEN model to show that the equilibrium action is implemented in a cost-minimizing manner when signals are independent. That is, given independence, Bernheim and Whinston's (1986) result on joint observations extends to disjoint observations in the LEN model. The underlying reason is that independence together with linear contracts and exponential utility imply so much "separability" that nothing is gained from collusion.

Now, the current model does not have the benefit of the same degree of separability. A complete analysis of the common agency problem in this setting is outside the scope of the paper but is planned for future research. However, a natural conjecture is that the equilibrium action is implemented at higher than minimum costs. Thus, the model has a source of distortion that is absent in Bernheim and Whinston (1986) and Holmström and Milgrom (1988).

8 Conclusion

This paper extends the canonical principal-agent model to allow the agent to pursue private, stochastic, and possibly non-monetary rewards. Conceptually, this way of "unpacking" leisure recognizes that rewards earned while not on the job are also endogenous. Hence, the principal manipulates not only the agent's effort on the job but also his "work-life balance" through the contract design.

The non-separability between rewards from labor income and other sources are at the root of the paper's economic insights. For instance, higher base utility at work reduces the incentive to pursue outside rewards. To an outside observer, the agent may now appear more "intrinsically motivated" as he can be induced to work hard on the job with flatter extrinsic incentives. It also explains why the agent may earn rent above his reservation utility and why implementations costs may be non-monotonic in effort on the job.

At the technical level, it is also this non-separability that represents the main challenge. Thus, the paper's technical contribution is to justify the FOA in environments with private rewards without assuming separability. Recall that separability is implicit in the LEN model, for instance. Thus, the paper's results demonstrate that non-separability have important economic implications.

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Appendix

Proof of Lemma 1. Integration by parts with respect to x_2 yields

$$EU(a_1,a_2) = \int \left(v(w(x_1),\overline{x}_2) - \int v_2(w(x_1),x_2)G^2(x_2|a_2)dx_2 \right) g^1(x_1|a_1)dx_1 - c(a_1,a_2). \tag{22}$$

Since $v_2 > 0$, Assumption A3 implies that the first term is strictly concave in a_2. The second term is weakly concave in a_2 by Assumption A4. Thus, $EU_{22} < 0$.

Assuming the contract is regular, another round of integrating by parts, this time with respect to x_1, yields

$$\begin{aligned} EU(a_1,a_2) \quad = \quad & -\int v_1(w(x_1),\overline{x}_2)w'(x_1)G^1(x_1|a_1)dx_1 - \int v_2(w(\overline{x}_1),x_2)G^2(x_2|a_2)dx_2 \\ & + \int\int v_{12}(w(x_1),x_2)w'(x_1)G^1(x_1|a_1)G^2(x_2|a_2)dx_1dx_2 \\ & + v(w(\overline{x}_1),\overline{x}_2) - c(a_1,a_2). \end{aligned} \tag{23}$$

Thus,

$$EU_{12}(a_1,a_2) = \int\int v_{12}(w(x_1),x_2)w'(x_1)G^1_{a_1}(x_1|a_1)G^2_{a_2}(x_2|a_2)dx_1dx_2 - c_{12}(a_1,a_2).$$

Recalling that $G^i_{a_i}(x_i|a_i) < 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$, $i = 1,2$, the last two parts of Assumption A4 (substitutes), $v_{12} < 0$ and $c_{12} \geq 0$, pull in the same direction. In particular, since $v_{12}(w(x_1),x_2)w'(x_1) \geq 0$ always, with strict inequality on a set of positive measure, it holds that $EU_{12}(a_1,a_2) < 0$. A similar argument proves that $EU_{11}(a_1,a_2) < 0$ if the contract is regular.

Since $v_1, v_2 > 0 > v_{12}$ and $G^1(x_1|a_1)$, $G^2(x_2|a_2)$, $G^1(x_1|a_1)G^2(x_2|a_2)$, and $c(a_1,a_2)$ are all convex in $(a_{1,}a_2)$, it follows from (23) that $EU(a_1,a_2)$ is concave because it is the sum of concave functions. To prove that $EU_{11}EU_{22} - EU_{12}^2 > 0$ when $w(x_1)$ is regular, let $P(a_1,a_2)$ denote the first line in (23) and let $Q(a_1,a_2)$ denote the remainder, such that $EU = P + Q$. Note that $P_{11}, P_{22} < 0$ but $P_{12} = 0$.

Similarly, $Q_{11}, Q_{22} < 0$ and by concavity $Q_{11}Q_{12} - Q_{12}^2 \geq 0$. Now,

$$\begin{aligned} EU_{11}EU_{22} - EU_{12}^2 &= \left[P_{11}P_{22} - P_{12}^2\right] + \left[Q_{11}Q_{22} - Q_{12}^2\right] + \left[P_{11}Q_{22} + P_{22}Q_{11} - 2P_{12}Q_{12}\right] \\ &= \left[P_{11}P_{22}\right] + \left[Q_{11}Q_{22} - Q_{12}^2\right] + \left[P_{11}Q_{22} + P_{22}Q_{11}\right] > 0, \end{aligned}$$

since the first and third terms are strictly positive and the second term is non-negative. This implies strict concavity. ■

Proof of Proposition 1. Since $w(x_1)$ is regular, Lemma 1 implies that $EU_{11} < 0$, $EU_{22} < 0$, and $EU_{12} < 0$. Moreover, $EU_{11}EU_{22} - EU_{12}^2 > 0$, or

$$\frac{-EU_{12}}{EU_{22}} > \frac{-EU_{11}}{EU_{12}}. \tag{24}$$

In (a_1, a_2) space, the curves along which $EU_1 = 0$ and $EU_2 = 0$ have slope

$$\frac{da_2}{da_1}_{|EU_1=0} = \frac{-EU_{11}}{EU_{12}} < 0 \text{ and } \frac{da_2}{da_1}_{|EU_2=0} = \frac{-EU_{12}}{EU_{22}} < 0.$$

Given contract $w(x_1)$, the optimal interior action (a_1^*, a_2^*) is found where these two curves intersect. By (24) the curve where $EU_1 = 0$ crosses the curve where $EU_2 = 0$ exactly once, from above.

In the following, let EU^ε denote the agent's expected utility when the contract is $\widehat{w}_\varepsilon(x_1)$ instead of $w(x_1)$. Note that $\widehat{w}_\varepsilon(x_1)$ is also a regular contract. Next, note that by design of $\widehat{w}_\varepsilon(x_1)$, $EU_1^\varepsilon = 0$ at (a_1^*, a_2^*). That is, both the $EU_1 = 0$ curve and the $EU_1^\varepsilon = 0$ curve go through the point (a_1^*, a_2^*). However, by (22) in Lemma 1,

$$\begin{aligned} EU_2(a_1, a_2) &= -\int \left(\int v_2(w(x_1), x_2) G_{a_2}^2(x_2|a_2) dx_2 \right) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) \\ &< -\int \left(\int v_2(\widehat{w}_\varepsilon(x_1), x_2) G_{a_2}^2(x_2|a_2) dx_2 \right) g^1(x_1|a_1) dx_1 - c_2(a_1, a_2) \\ &= EU_2^\varepsilon(a_1, a_2), \end{aligned}$$

for all (a_1, a_2) since $v_{12} < 0$, $\widehat{w}_\varepsilon(x_1) < w(x_1)$, and $G_{a_2}^2 < 0$. Then, $EU_{22}^\varepsilon < 0$ implies that the curve where $EU_2^\varepsilon = 0$ under $\widehat{w}_\varepsilon(x_1)$ must lie above the curve where $EU_2 = 0$ under $w(x_1)$. Thus, it can immediately be ruled out that $a_1' = a_1^*$.

Imagine that $a_1^{'} > a_1^*$. Then, since $EU_1^\varepsilon = 0$ crosses the curve where $EU_2^\varepsilon = 0$ from above at (a_1', a_2') under $\widehat{w}_\varepsilon(x_1)$, it must hold that the $EU_1^\varepsilon = 0$ curve lies above the $EU_2^\varepsilon = 0$ curve for any $a_1 < a_1'$. Combined with the observation in the last paragraph, this implies that $EU_1^\varepsilon(a_1^*, a_2^*) \neq 0$, which is a contradiction. Thus, $a_1^{'} < a_1^*$. As the $EU_2^\varepsilon = 0$ curve under $\widehat{w}_\varepsilon(x_1)$ lies above the $EU_2 = 0$ curve under $w(x_1)$, it then follows that $a_2' > a_2^*$. ■

Proof of Proposition 2. From the proof of Proposition 1, the curve where $EU_2 = 0$ under contract $\widetilde{w}(x_1)$ lies above its counterpart under $w(x_1)$, in (a_1, a_2) space. For $a_1' = a_1^*$ to be optimal, the curve where $EU_1 = 0$ must cross the $EU_2 = 0$ curve at a_1^*. This in turn implies that $a_2' > a_2^*$. Since $EU_{12} < 0$, it is then necessary that $EU_1(a_1^*, a_2^*) > 0$ under $\widetilde{w}(x_1)$. ■

Proof of Lemma 2. First,

$$V_1(w, a_2) = \int v_1(w, x_2) g^2(x_2|a_2) dx_2.$$

Assumption A4 (substitutes) implies that $V(w, a_2)$ is strictly increasing and strictly concave in w, or $V_1(w, a_2) > 0 > V_{11}(w, a_2)$, just like a standard utility function. Moreover, A2 and A4 together imply that

$$V_{12}(w, a_2) = \int v_1(w, x_2) g^2_{a_2}(x_2|a_2) dx_2 < 0.$$

The reason is that $v_1(w, x_2)$ is strictly decreasing in x_2 (A4) and that an increase in a_2 leads $G^2(x_2|a_2)$ to become stochastically stronger in the sense of first order stochastic dominance (A2). Similarly, Assumption A4 together with A2 and A3 (LOCC) imply that $V_2(w, a_2) > 0$ and $V_{22}(w, a_2) < 0$, respectively. This proves the first two parts of the lemma.

For the third part, note that the term under the integration sign in $V_1(w, a_2)$ is, by Assumptions A2 (MLRP) and A5 (Log-supermodularity), log-supermodular in *all* (three) arguments, (w, x_2, a_2). In this case, log-supermodularity is preserved under integration so $V_1(w, a_2)$ is log-supermodular in its (two) arguments (w, a_2); see e.g. Athey (2002, p. 193). This, proves the third part of the lemma. The fourth part follows from carrying out the differentiation in (9) and using the sign restrictions implied by parts 1 and 2. ■

Proof of Lemma 3. Given $\mu_2 \leq 0$, $V_{11} < 0$ and (9) imply that the right hand side of (10) is strictly increasing in w (the derivative is strictly positive). Thus, for each x_1 there is at most one solution to (10), $w(x_1)$. Differentiability now follows from the differentiability of all the components in (10) and the fact that the right hand side is strictly increasing in w. If $\mu_1 > 0$, Assumption A2 (MLRP) implies that the left hand side is non-decreasing in x_1 and strictly increasing on a subset of positive measure. Hence, the contract is regular. ■

Proof of Proposition 6. For interior actions, the proof is in the text preceding the proposition. For actions involving $a_2 = \overline{a}_2$, incentive compatibility requires $EU_2 \geq 0$. Thus, the contract's certainty equivalent must be at most $CE_{-V_2}(a_1, \overline{a}_2)$. However, if the action is outside $T(\overline{u})$ then such a contract violates the participation constraint (remember that $CE_V = CE_{-V_2}$ in the multiplicative model). ■

Proof of Theorem 1. Any solution to the doubly-relaxed problem must take the form in (10), with $\mu_1 \geq 0 \geq \mu_2$. However, wages are constant if $\mu_1 = 0$. Then, $EU_1 = -c_1 < 0$, which violates the doubly-relaxed constraints. Hence, $\mu_1 > 0$ and so $EU_1 = 0$. By Lemma 3, any solution involves a regular contract. By Lemma 1, the agent's problem is concave. The contract is then incentive compatible if $EU_1 = EU_2 = 0$ at the intended action, which holds if $\mu_1 > 0 > \mu_2$.

Thus, the next step establishes that $\mu_2 < 0$. If an interior a_2 is optimal in the doubly-relaxed problem then (12) must hold. By Assumption P2, $B_2 \leq 0$. By Lemma 1, it holds that $EU_{12} < 0$ given the contract is regular. Since $\lambda EU_2 \leq 0$, the first three terms in (12) are thus strictly negative. As $EU_{22} < 0$, it is therefore necessary that $\mu_2 < 0$. Hence, $EU_1 = EU_2 = 0$.

By assumption, any second-best action is interior. Thus, any solution to the unrelaxed problem must satisfy $EU_1 = EU_2 = 0$, which implies that it is feasible in the doubly-relaxed problem. However, it has just been shown that any interior solution to the doubly-relaxed problem is feasible in the unrelaxed problem. Hence, the solutions to the unrelaxed and doubly-relaxed problems coincide. Finally, the set of feasible contracts is obviously larger in the doubly-relaxed problem than in the relaxed problem. Then, as the solution to the doubly-relaxed problem involves an interior action, the solution is also feasible in the relaxed problem, which must then identify the exact same solution. ■

Proof of Proposition 4. First,

$$\begin{aligned}\frac{\partial \overline{U}(a_1,a_2)}{\partial a_2} &= V_1(CE_{-V_2},a_2)\frac{\partial CE_{-V_2}}{\partial a_2} + V_2(CE_{-V_2},a_2) - c_2(a_1,a_2) \\ &= V_1(CE_{-V_2},a_2)\frac{\partial CE_{-V_2}}{\partial a_2} < 0,\end{aligned}$$

where the last equality follows from L-IC$_2$. It is readily confirmed that $\overline{U}(a_1,a_2)$ is strictly decreasing in a_1. Thus, if $\overline{U}(a_1,a_2) \geq \overline{u}$ then $\overline{U}(a_1',a_2') \geq \overline{u}$ whenever $(a_1',a_2') \leq (a_1,a_2)$. This proves the second part of the proposition. The last part follows by definition of $T(\overline{u})$. ■

Proof of Theorem 2. The reduced problem produces a contract of the form in (10), but with $\lambda = 0$. Thus, there exists at least one value of x_1 for which the left hand side is zero. Any solution to (10) then requires $\mu_2 < 0$. Hence, the right hand side is strictly increasing in w, by A4 and A5. If $\mu_1 \leq 0$, the contract is weakly decreasing, by A2. In this case, $EU_1 < 0$ which violates one of the constraints of the reduced problem. Hence, $\mu_1 > 0$. In summary, the contract is regular by Lemma 3, since $\lambda = 0$ and $\mu_1 > 0 > \mu_2$. By Lemma 1, the contract is incentive compatible. By the assumption imposed on $T(\overline{u})$, (P) is satisfied. Hence, the contract is feasible in both the relaxed and the unrelaxed problems, which it must then solve given the second-best action is interior. ■

Proof of Proposition 7. Since the contract solves the reduced problem it must be regular, by the argument in Theorem 2. Since the contract also solves the relaxed problem, it must satisfy (P). Hence, the contract is feasible in the unrelaxed problem, which it must then solve as the second-best action is interior.
■